

Advanced Programme

Mathematics IEB

BOOK 1

Marilyn Buchanan, Anne Eadie, Carl Fourie, Noleen Jakins
& Ingrid Zlobinsky-Roux

GRADE

10-12

IEB

SOLUTIONS



THE
ANSWER
SERIES *Your Key to Exam Success*



GRADE

10-12

IEB

3-in-1

Advanced Programme Mathematics IEB

Book 1: Solutions

Marilyn Buchanan, Anne Eadie, Carl Fourie, Noleen Jakins
& Ingrid Zlobinsky-Roux

THIS CLASS TEXT & STUDY GUIDE INCLUDES

Full Solutions to Exercises and Exam questions:

- Calculus
- Algebra

E-book
available 



CONTENTS

Solutions to Book 1: Compulsory Modules



GRADE 10

Chapter	Module	Title	Pg. no.
1	Algebra	Products, Factors & Compound fractions.....	1
2	Algebra	Absolute values	8
3	Algebra	Complex numbers	14
4	Calculus	Area under curves.....	24

GRADE 11

Chapter	Module	Title	Pg. no.
5	Algebra	Partial fractions.....	26
6	Algebra	Equations & Inequalities	28
7	Calculus	Functions	35
8	Algebra	Absolute Value equations, Graphs & Inequalities	41
9	Calculus	Limits & Continuity	58
10	Calculus	Derivatives	63
11	Calculus	Trigonometry.....	64
12	Calculus	Integrals & Integration.....	70

GRADE 12

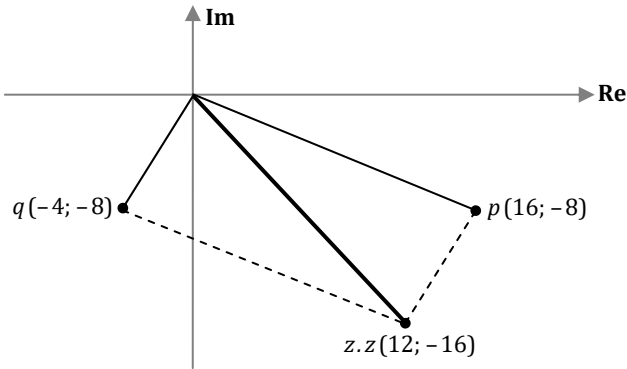
Chapter	Module	Title	Pg. no.
13	Algebra	Mathematical Induction.....	75
14	Algebra	e & \ln	86
15	Calculus	Further Derivatives.....	94
16	Calculus	Continuity & Differentiability.....	106
17	Calculus	Derivative applications	110
18	Calculus	Integration.....	125



Chapter 3: Exercise 3.3 & Gr 10 Complex Numbers Exam

(j) $z^2 = z \cdot z$
 $= (-4 + 2i)(-4 + 2i)$
 $= -4(-4 + 2i) + 2i(-4 + 2i)$
 $= (16 - 8i) + (-8i + 4i^2)$
 $= (16 - 8i) + (-4 - 8i)$
 $= 12 - 16i$

The product of two complex numbers is rewritten as the sum of two different numbers. The result is the diagonal from the origin of the rectangle. The product of a complex conjugate pair is always real.



Gr 10 Complex Numbers Exam Solutions

(Questions - p. 40 in Book)

1. $x^3 - 1 = (x - 1)(x^2 + x + 1) = 0$
 $x = 1$ or $x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{3}i}{2}$

2. $(a + 3i)bi = (-11 - 13i)(2 - 5i)$
 $\therefore abi + 3bi^2 = -22 + 55i - 26i + 65i^2$
 $\therefore -3b + abi = -87 + 29i$
 $\therefore -3b = -87$ and $ab = 29$
 $\therefore a = 1, b = 29$

3. $2z - iw = 2(5 - 2i) - i(6i - 1)$
 $= 10 - 4i - 6i^2 + i$
 $= 16 - 3i$

4. $\frac{a + bi}{a - bi} \times \frac{a + bi}{a + bi} = \frac{a^2 + 2abi + b^2i^2}{a^2 - b^2i^2} = \frac{a^2 - b^2 + 2abi}{a^2 + b^2}$
real = $\frac{a^2 - b^2}{a^2 + b^2}$

5. Let $\alpha = q + \sqrt{3}i$, and its conjugate $\beta = q - \sqrt{3}i$
 $\alpha + \beta = 2q = -\frac{b}{a} = -(-2) \quad \therefore q = 1$
 $\alpha\beta = q^2 - 3i^2 = q^2 + 3 = \frac{c}{a} = p \quad \therefore p = 4$

6. (a) $b^2 - 4ac < 0$ for non-real roots
 $\therefore p^2 - 4p(1) < 0$
 $\therefore p(p - 4) < 0$
 $\therefore 0 < p < 4$

(b) $i + i^2 + i^3 + \dots + i^{2017}$
 $= (i - 1 - i + 1) + (i - 1 - i + 1) + \dots i = i$

7. $\frac{a + bi}{-b + ai} \times \frac{-b - ai}{-b - ai} = \frac{-ab - b^2i - a^2i - abi^2}{b^2 - a^2i^2}$
 $= \frac{-ab - i(b^2 + a^2) + ab}{b^2 + a^2}$
 $= -i$

8. (a) $m - 2n^* = (4 + 2i) - 2(-2 + i) = 4 + 2i + 4 - 2i = 8$

(b) $\frac{m}{n} = \frac{4 + 2i}{-2 - i} \times \frac{-2 + i}{-2 + i} = \frac{-8 + 2i^2}{4 - i^2} = \frac{-8 - 2}{5} = -2$

9. (a) $x^2 + 8x + 16 + 9 = (x + 4)^2 - 9i^2$
 $= (x + 4 - 3i)(x + 4 + 3i)$

(b) Let $\alpha = 2 + 3i$, and its conjugate $\beta = 2 - 3i$

$\alpha + \beta = 4 = -\frac{b}{a}$ and $\alpha\beta = 13 = \frac{c}{a}$

If $ax^2 + bx + c = 0$ then $a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0$

$a(x^2 - 4x + 13) = 0$ will produce the desired roots.

10. (a) $b^2 - 4ac = (-4)^2 - 4(1)(-8) = 48$

(b) **two real, irrational roots**

(c) **12** by trial and error or

$x^2 - 4x - 8 + t = 0$

$b^2 - 4ac = (-4)^2 - 4(-8 + t) = 0$ for one real root

Hence, $t = 12$.

11. $r^2 = 36 + 4 = 40$

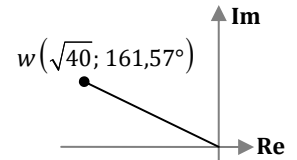
$\therefore r = \sqrt{40} \approx 6,32$

$\tan \theta = -\frac{2}{6}$

$(-6; 2)$ lies in second quadrant

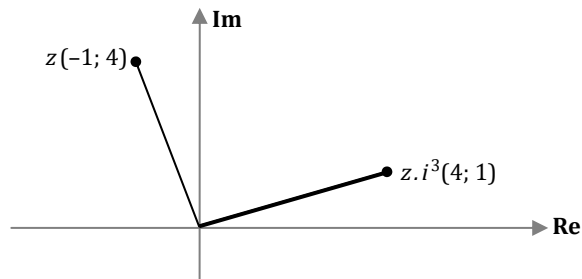
$\theta = 180^\circ - 18,43^\circ = 161,57^\circ$

$w(\sqrt{40}; 161,57^\circ)$

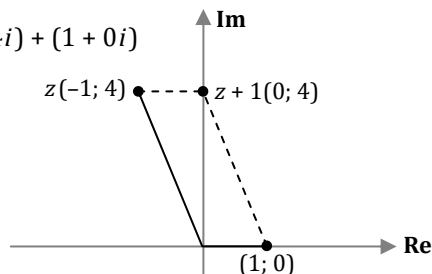


12. (a) $z \cdot i^3 = z \cdot (-i)$
 $= -z \cdot i$
 $= -(-1 + 4i)i$
 $= -(-1i + 4i^2)$
 $= i - 4i^2$
 $= 4 + i$

Multiplying by i rotates z by 90° anticlockwise about the origin. Multiplying by (-1) rotates the result by 180° about the origin. Hence the overall result is z rotated by 270° anticlockwise about the origin.



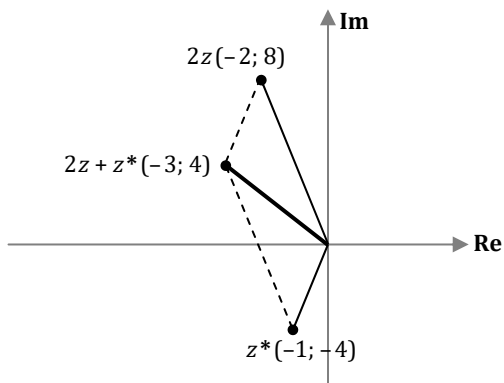
(b) $z + 1$
 $= (-1 + 4i) + (1 + 0i)$
 $= 4i$



The sum of two complex numbers is the diagonal from the origin of the parallelogram.

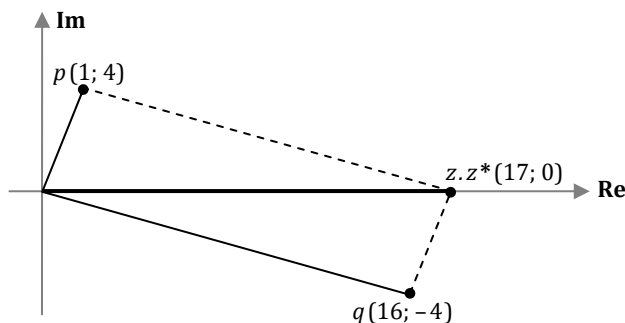
(c) $2z + z^* = (-2 + 8i) + (-1 - 4i)$
 $= -3 + 4i$

The sum of two complex numbers is the diagonal from the origin of the parallelogram.



(d) $z.z^* = (-1 + 4i)(-1 - 4i)$
 $= -1(-1 - 4i) + 4i(-1 - 4i)$
 $= (1 + 4i) + (-4i - 16i^2)$
 $= (1 + 4i) + (16 - 4i)$
 $= 17$

The product of two complex numbers is rewritten as the sum of two different numbers. The result is the diagonal from the origin of the rectangle. The product of a complex conjugate pair is always real.



13. $3p + 3qi + pi + qi^2 = -4 + 2i$
 $\therefore (3p - q) + (3q + p)i = -4 + 2i$
 $\therefore 3p - q = -4 \quad \text{and} \quad p + 3q = 2$
 $\therefore q = 1 \quad \text{and} \quad p = -1$

solve simultaneously

14. For $x^2 + kx + t = 0$, $-\frac{b}{a} = -\frac{k}{1}$ and $\frac{c}{a} = \frac{t}{1}$.
 $(a + bi)(a - bi) = a^2 - b^2i^2 = a^2 + b^2 = \frac{t}{1} = t$
 $(a + bi) + (a - bi) = 2a = -\frac{k}{1}$ so $2a + k = 0$.

Exercise 3.4

(Questions - p. 43 in Book)

1. (a) $2x^2 - \frac{13x}{3} + 2 = \frac{6x^2 - 13x + 6}{3}$
 $= \frac{(3x - 2)(2x - 3)}{3} = 0$

$\therefore x = \frac{2}{3} \quad \text{or} \quad \frac{3}{2}$

(b) $\frac{2x^2}{3} + \frac{x}{3} - 1 = \frac{2x^2 + x - 3}{3} = \frac{(2x + 3)(x - 1)}{3} = 0$

$\therefore x = -\frac{3}{2} \quad \text{or} \quad 1$

(c) $4x^4 + 3x^2 - 1 = (4x^2 - 1)(x^2 + 1) = 0$

$\therefore x = \pm \frac{1}{2} \quad \text{or} \quad x = \pm i$

(d) $x^4 - x^2 - 12 = (x^2 - 4)(x^2 + 3) = 0$

$x = \pm 2 \quad \text{or} \quad x = \pm \sqrt{3}i$

(e) $x^4 + 3x^2 - 28 = (x^2 - 4)(x^2 + 7) = 0$

$x = \pm 2 \quad \text{or} \quad x = \pm \sqrt{7}i$

(f) $3x^4 + 25x^2 - 18 = (3x^2 - 2)(x^2 + 9) = 0$

$x = \pm \sqrt{\frac{2}{3}} \quad \text{or} \quad x = \pm 3i$

2. (a) $x^3 + x^2 - x - 1 = x^2(x + 1) - (x + 1)$
 $= (x + 1)(x^2 - 1) = 0$

$\therefore x = \pm 1$

(b) $x^3 + 4x^2 + 9x + 36 = x^2(x + 4) + 9(x + 4)$
 $= (x + 4)(x^2 + 9) = 0$

$\therefore x = -4, \quad x = \pm 3i$

(c) $x^3 + 2x^2 + 6x + 12 = x^2(x + 2) + 6(x + 2)$
 $= (x + 2)(x^2 + 6) = 0$

$\therefore x = -2, \quad x = \pm \sqrt{6}i$

Chapter 5: Exercise 5.2 & 5.3

$$7. \frac{9x-24}{x^2-6x+8} = \frac{9x-24}{(x-4)(x-2)} \equiv \frac{A}{x-4} + \frac{B}{x-2}$$

$$A = \frac{12}{2} = 6 \quad B = \frac{-6}{-2} = 3$$

$$\frac{9x-24}{x^2-6x+8} = \frac{6}{x-4} + \frac{3}{x-2}$$

$$8. \frac{1}{x^2-2x-8} = \frac{1}{(x-4)(x+2)} \equiv \frac{A}{x-4} + \frac{B}{x+2}$$

$$A = \frac{1}{6} \quad B = \frac{1}{-6}$$

$$\frac{1}{x^2-2x-8} = \frac{1}{6(x-4)} - \frac{1}{6(x+2)}$$

$$9. \frac{23x-1}{6x^2+x-1} = \frac{23x-1}{(3x-1)(2x+1)} \equiv \frac{A}{3x-1} + \frac{B}{2x+1}$$

$$A = \frac{\frac{23}{3}-1}{\frac{2}{3}+1} = 4 \quad B = \frac{\frac{-23}{2}-1}{\frac{-3}{2}-1} = 5$$

$$\frac{23x-1}{6x^2+x-1} = \frac{4}{3x-1} + \frac{5}{2x+1}$$

$$10. \frac{6x^2-22x+18}{(x-1)(x-2)(x-3)} \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$A = \frac{2}{2} = 1 \quad B = \frac{-2}{-1} = 2 \quad C = \frac{6}{2} = 3$$

$$\frac{6x^2-22x+18}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3}$$

Exercise 5.3

(Questions - p. 57 in Book)

$$1. (a) \frac{3x+4}{(x+3)^2} = \frac{A}{x+3} + \frac{B}{(x+3)^2}$$

$$\therefore 3x+4 \equiv A(x+3) + B$$

$$\text{Let } x = -3: \therefore -5 = B \quad \Rightarrow B = -5$$

$$\text{Let } x = 0: \therefore 4 = 3A - 5 \quad \Rightarrow A = 3$$

$$\therefore \frac{3x+4}{(x+3)^2} = \frac{3}{x+3} - \frac{5}{(x+3)^2}$$

$$(b) \frac{x^2-7x+12}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\therefore x^2-7x+12 \equiv A(x-2)^2 + Bx(x-2) + Cx$$

$$\text{Let } x = 0: \therefore 12 = 4A \quad \Rightarrow A = 3$$

$$\text{Let } x = 2: \therefore 2 = 2C \quad \Rightarrow C = 1$$

$$\text{Let } x = 1: \therefore 6 = 3 + B(-1) + 1 \quad \Rightarrow B = -2$$

$$\therefore \frac{x^2-7x+12}{x(x-2)^2} = \frac{3}{x} - \frac{2}{x-2} + \frac{1}{(x-2)^2}$$

$$(c) \frac{8x-12}{(x+3)(x^2-6x+9)} = \frac{8x-12}{(x+3)(x-3)^2} = \frac{A}{x+3} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$\therefore 8x-12 \equiv A(x-3)^2 + B(x+3)(x-3) + C(x+3)$$

$$\text{Let } x = -3: \therefore -36 = 36A \quad \Rightarrow A = -1$$

$$\text{Let } x = 3: \therefore 12 = 6C \quad \Rightarrow C = 2$$

$$\text{Let } x = 0: \therefore -12 = -9 - 9B + 6 \quad \Rightarrow B = 1$$

$$\therefore \frac{8x-12}{(x+3)(x^2-6x+9)} = \frac{-1}{x+3} + \frac{1}{x-3} + \frac{2}{(x-3)^2}$$

$$(d) \frac{2x^2-9x+16}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\therefore 2x^2-9x+16 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$$

$$\text{Let } x = -1: \therefore 27 = 9A \quad \Rightarrow A = 3$$

$$\text{Let } x = 2: \therefore 6 = 3C \quad \Rightarrow C = 2$$

$$\text{Let } x = 0: \therefore 16 = 12 - 2B + 2 \quad \Rightarrow B = -1$$

$$\therefore \frac{2x^2-9x+16}{(x+1)(x-2)^2} = \frac{3}{x+1} - \frac{1}{x-2} + \frac{2}{(x-2)^2}$$

$$(e) \frac{2x^2+1}{x^2(x^2-2x+1)} = \frac{2x^2+1}{x^2(x-1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$\therefore 2x^2+1 \equiv Ax(x-1)^2 + B(x-1)^2 + Cx^2(x-1) + Dx^2$$

$$\text{Let } x = 1: \therefore 3 = D$$

$$\text{Let } x = 0: \therefore 1 = B$$

$$\text{Let } x = -1: \therefore 3 = -4A + 4 - 2C + 3$$

$$\therefore 4A + 2C = 4 \quad \dots \textcircled{1}$$

$$\text{Let } x = 2: \therefore 9 = 2A + 1 + 4C + 12$$

$$\therefore 2A + 4C = -4 \quad \dots \textcircled{2}$$

Solve equations $\textcircled{1}$ and $\textcircled{2}$ simultaneously, using the calculator:

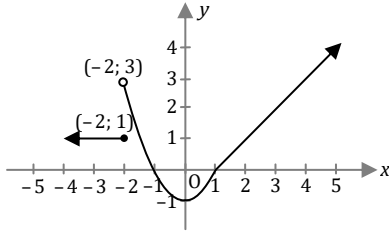
$$\therefore A = 2 \text{ and } C = -2$$

$$\therefore \frac{2x^2+1}{x^2(x^2-2x+1)} = \frac{2}{x} + \frac{1}{x^2} - \frac{2}{x-1} + \frac{3}{(x-1)^2}$$



Chapter 7: Exercise 7.2 & 7.3

4. (a)



(b) Domain: $x \in \mathbb{R}$ Range: $y \geq -1$

(c) $f(-3) = 1$ $f(-1) = 0$ $f(3) = 2$

(d) (i) **No solution** (ii) $x = 4$

(iii) $x \leq -2$ or $x = 2$

5. $f(x) = x + \frac{1}{x}, x \neq 0$

$$y = \frac{x^2 + 1}{x}$$

$$\therefore yx = x^2 + 1$$

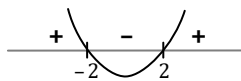
$$\therefore 0 = x^2 - yx + 1 \quad \text{quadratic equation}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+y \pm \sqrt{y^2 - 4}}{2}$$

Restrictions placed on the range such that the domain is real:

$$y^2 - 4 \geq 0$$

$$\therefore (y - 2)(y + 2) \geq 0$$



$$\therefore -\infty < y \leq -2 \quad \text{or} \quad 2 \leq y < \infty$$



Exercise 7.3

(Questions – p. 75 in Book)

1. (a) $f(g(5))$
 $g(5) = 2$
 $f(2) = 4$
 $\therefore f(g(5)) = 4$

Outer function

(c) $f(g(x))$

Inner function

$$= (g(x))^2$$

$$= (x - 3)^2$$

Outer function

(e) $f(f(x))$

Inner function

$$= (x^2)^2$$

$$= x^4$$

2. (a) $f(g(9))$
 $= f(2)$
 $= 2$

(c) $f(g(0))$
 $= f(-1)$
 $= 1$

(e) $f(g(-4))$
undefined

(b) $g(f(5))$
 $f(5) = 25$
 $g(25) = 22$
 $\therefore g(f(5)) = 22$

Outer function

(d) $g(f(x))$

Inner function

$$= (f(x)) - 3$$

$$= (x^2) - 3$$

$$= x^2 - 3$$

Outer function

(f) $g(g(x))$

Inner function

$$= (x - 3) - 3$$

$$= x - 6$$

(b) $g(f(9))$
 $= g(9)$
 $= \sqrt{9} - 1 = 2$

(d) $g(f(0))$
 $= g(0)$
 $= -1$

(f) $g(f(-4))$
 $= g(4)$
 $= \sqrt{4} - 1$
 $= 1$

(g) $f(g(x))$

$$= \begin{cases} (g(x)) & \text{if } (g(x)) \geq 0 \\ -(g(x)) & \text{if } (g(x)) < 0 \end{cases}$$

$$= \begin{cases} (\sqrt{x} - 1) & \text{if } \sqrt{x} - 1 \geq 0 \\ -(\sqrt{x} - 1) & \text{if } \sqrt{x} - 1 < 0 \end{cases}$$

$$\begin{cases} \sqrt{x} \geq 1 \\ x \geq 1 \end{cases}$$

$$= \begin{cases} \sqrt{x} - 1 & \text{if } x \geq 1 \\ -\sqrt{x} + 1 & \text{if } x < 1 \end{cases}$$

(h) $g(f(x))$

$$= \begin{cases} \sqrt{(x)} - 1 & \text{if } x \geq 0 \\ \sqrt{-(x)} - 1 & \text{if } x < 0 \end{cases}$$

3. (a) $f(x) = 3x - 2$ $g(x) = 2 - x$
 $f(g(x)) = 3(g(x)) - 2$ $g(f(x)) = 2 - (f(x))$
 $= 3(2 - x) - 2$ $= 2 - (3x - 2)$
 $= 4 - 3x$ $= 4 - 3x$

Domain: $x \in \mathbb{R}$

Domain: $x \in \mathbb{R}$

(b) $f(x) = x^2$ $g(x) = x + 2$
 $f(g(x)) = (g(x))^2$ $g(f(x)) = (f(x)) + 2$
 $= (x + 2)^2$ $= x^2 + 2$

Domain: $x \in \mathbb{R}$

Domain: $x \in \mathbb{R}$

(c) $f(x) = x^2 - 4$ $g(x) = \sqrt{x}$
 $f(g(x)) = (\sqrt{x})^2 - 4$ $g(f(x)) = \sqrt{x^2 - 4}$
 $= x - 4$

Domain $g(x)$: $x \geq 0$

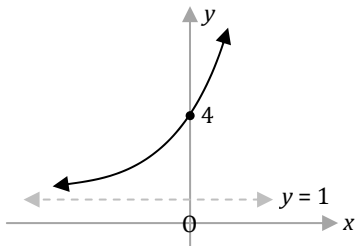
Domain: $x^2 - 4 \geq 0$

Domain $f \circ g$: $x \geq 0$

$\therefore x \leq -2$ or $x \geq 2$

Chapter 8: Exercise 8.3 & 8.4

7. $y = |3^{x+1} + 1| = 3^{x+1} + 1$



Note:
 $|3^{x+1} + 1|$
 $= 3^{x+1} + 1$
 for all real values of x

Domain: $x \in \mathbb{R}$
 Range: $y > 1$

8. $y = |2^{1+x-1} - 3| = |2^x - 3|$

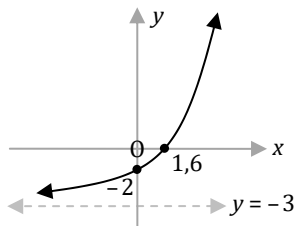
y -intercept ($x = 0$): $y = |1 - 3| = 2$

x -intercept ($y = 0$): $2^x = 3$
 $\therefore x \approx 1,6$

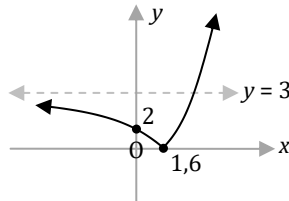


SOLVE:
 Appendix 1

$y = 2 \cdot 2^{x-1} - 3$:



$y = |2 \cdot 2^{x-1} - 3|$:



Domain: $x \in \mathbb{R}$
 Range: $y \geq 0$

Exercise 8.4

(Questions - p. 88 in Book)

1. (a) $y = -|x|^2 + 2|x| + 15$

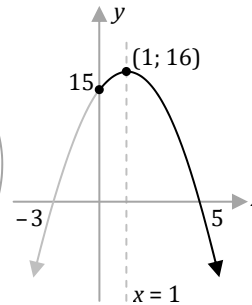
For $x \geq 0$:

$y = -x^2 + 2x + 15$
 $\therefore y = -(x^2 - 2x - 15)$
 $\therefore y = -(x+3)(x-5)$

y -intercept: $y = 15$

x -intercept(s): $x = 5$; $x \neq -3$

Turning point: $(1; 16)$



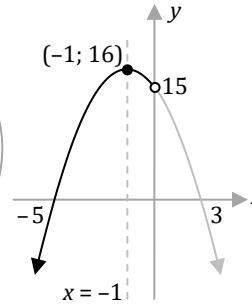
For $x < 0$:

$y = -x^2 - 2x + 15$
 $\therefore y = -(x^2 + 2x - 15)$
 $\therefore y = -(x+5)(x-3)$

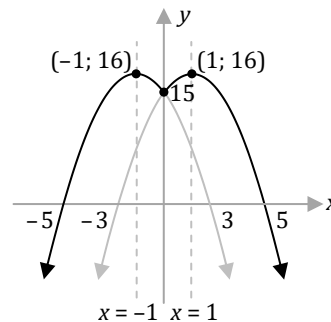
y -intercept: $y = 15$

x -intercept(s): $x = -5$; $x \neq 3$

Turning point: $(-1; 16)$



\therefore The graph of $y = -|x|^2 + 2|x| - 15$
 for ALL values of x :



Domain: $x \in \mathbb{R}$
 Range: $y \leq 16$

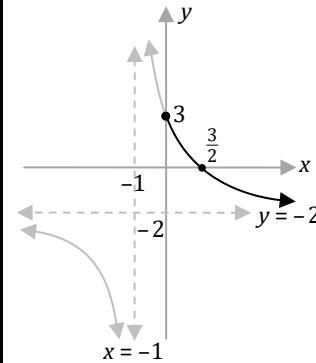
In examples (a) \rightarrow (d), the graph of $y = f(|x|)$ for $x \geq 0$ can be reflected in the y -axis to obtain the other part (where $x < 0$).



(b) $y = \frac{5}{|x|+1} - 2$

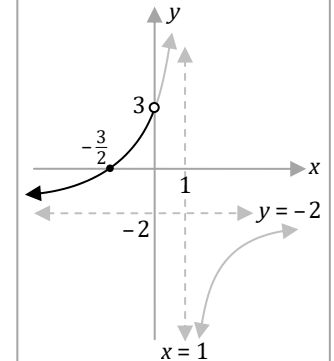
For $x \geq 0$:

$y = \frac{5}{x+1} - 2$

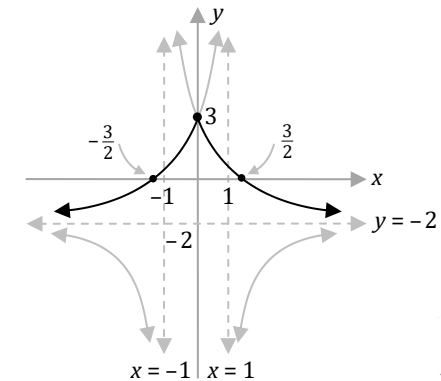


For $x < 0$:

$y = \frac{5}{-x+1} - 2$
 $\therefore y = \frac{-5}{x-1} - 2$



\therefore The graph of $y = \frac{5}{|x|+1} - 2$ for ALL values of x :



Domain: $x \in \mathbb{R}$
 Range: $-2 < y \leq 3$

Note that the final graphs in (a), (b), (c) and (d) are all symmetrical about the y -axis.



$$(e) f(x) = 3x - \frac{1}{\sqrt{x}} = 3x - x^{-\frac{1}{2}}$$

$$f'(x) = 3 + \frac{1}{2} \cdot x^{-\frac{3}{2}} = 3 + \frac{1}{2\sqrt{x^3}}$$

$$(f) f(x) = x^3 - 6x^2 + 9x - 4$$

$$f'(x) = 3x^2 - 12x + 9$$

$$(g) f(x) = \frac{x^3}{3} + x^2 - 5x + 1$$

$$f'(x) = x^2 + 2x - 5$$

$$(h) f(x) = \frac{x^2 - 4x}{x} = x - 4$$

$$f'(x) = 1$$

$$(i) f(x) = \frac{3x^2 + x - 1}{x} = 3x + 1 - x^{-1}$$

$$f'(x) = 3 + x^{-2} = 3 + \frac{1}{x^2}$$

2. (a) $y = 3x^3 + 5x^2 - 4x - 3$

$$\frac{dy}{dx} = 9x^2 + 10x - 4$$

(b) $g(x) = \frac{4x^2 - 1}{2x + 1}$

$$= \frac{(2x + 1)(2x - 1)}{2x + 1}$$

$$= 2x - 1; \quad x \neq -\frac{1}{2}$$

$$g'(x) = 2$$

3. (a) $D_x \left[x^2 - \frac{1}{x^3} \right]$

$$D_x [x^2 - x^{-3}]$$

$$= 2x + 3x^{-4}$$

$$= 2x + \frac{3}{x^4}$$

(b) $\frac{d}{dx} \left(\frac{1 + x^{\frac{3}{2}}}{x^2} \right)$

$$= \frac{d}{dx} \left(x^{-\frac{1}{2}} + x \right)$$

$$= -\frac{1}{2}x^{-\frac{3}{2}} + 1$$

$$= -\frac{1}{2x^{\frac{3}{2}}} + 1$$

$$= \frac{1}{2x^2} + 1$$

(c) $D_t \left[\frac{\sqrt{t} - 3t}{\sqrt{t}} \right]$

$$= D_t \left[1 - 3t^{\frac{1}{2}} \right]$$

$$= -\frac{3}{2}t^{-\frac{1}{2}}$$

$$= \frac{-3}{2\sqrt{t}}$$

(d) $\frac{d}{ds} \left(\frac{2s - s^2 + 3s^3}{s^2} \right)$

$$= \frac{d}{ds} (2s^{-1} - 1 + 3s)$$

$$= -2s^{-2} + 3$$

$$= \frac{-2}{s^2} + 3$$

(e) $f(x) = \frac{2x^3 - x^2 - 8x + 4}{x - 2}$

$$= \frac{x^2(2x - 1) - 4(2x - 1)}{x - 2}$$

$$= \frac{(2x - 1)(x - 2)(x + 2)}{x - 2}$$

$$= 2x^2 + 3x - 2, \quad x \neq 2$$

$$\therefore f'(x) = 4x + 3$$

Chapter 11: Grade 11 TRIGONOMETRY

Exercise 11.1

(Questions - p. 113 in Book)

1. (a) $\operatorname{cosec} \theta \cdot \tan \theta$

$$= \frac{1}{\sin \theta} \times \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta} = \sec \theta$$

(b) $\operatorname{cosec} \theta \cdot \frac{1}{\cot \theta}$

$$= \frac{1}{\sin \theta} \times \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta} = \sec \theta$$

(c) $\sec \theta \cdot \frac{1}{\tan \theta} \cdot \frac{1}{\operatorname{cosec} \theta}$

$$= \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} \times \sin \theta = 1$$

(d) $\cot \theta \cdot \operatorname{cosec} \theta \cdot \tan \theta$

$$= \frac{\cos \theta}{\sin \theta} \times \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1}{\sin \theta} = \operatorname{cosec} \theta$$

(e) $\tan \theta \cdot \cos \theta \cdot \cot \theta \cdot \sec \theta$

$$= \tan \theta \cdot \cos \theta \cdot \frac{1}{\tan \theta} \cdot \frac{1}{\cos \theta} = 1$$

2. (a) R.T.P.: $\tan A \cdot \operatorname{cosec}^2 A \cdot \cos^2 A = \cot A$

$$\text{LHS} = \frac{\sin A}{\cos A} \times \frac{1}{\sin^2 A} \times \cos^2 A$$

$$= \frac{\cos A}{\sin A} = \cot A = \text{RHS}$$

(b) R.T.P.: $\cot^2 A - \cos^2 A = \cot^2 A \cdot \cos^2 A$

$$\text{LHS} = \frac{\cos^2 A}{\sin^2 A} - \cos^2 A$$

$$= \frac{\cos^2 A - \cos^2 A \sin^2 A}{\sin^2 A}$$

$$= \frac{\cos^2 A(1 - \sin^2 A)}{\sin^2 A}$$

$$= \frac{\cos^2 A}{\sin^2 A} \times \cos^2 A = \cot^2 A \cdot \cos^2 A = \text{RHS}$$

Chapter 11: Exercise 11.3 & 11.4

$$\begin{aligned} \text{(b) Area segment} &= \frac{1}{2}(2)^2 \times \frac{\pi}{3} - \frac{1}{2}(2)^2 \sin \frac{\pi}{3} \\ &= \frac{2\pi}{3} - 2 \times \frac{\sqrt{3}}{2} \\ &= \frac{2\pi - 3\sqrt{3}}{3} \end{aligned}$$

$$\text{Area triangle} = \frac{1}{2}(2)^2 \sin \frac{\pi}{3} = \sqrt{3}$$

$$\begin{aligned} \text{Area of coin} &= 3 \times \left(\frac{2\pi - 3\sqrt{3}}{3} \right) + \sqrt{3} \\ &= 2\pi - 2\sqrt{3} \text{ cm}^2 \end{aligned}$$

Exercise 11.4

(Questions - p. 124 in Book)

$$1. \quad 2 \tan \left(x - \frac{\pi}{12} \right) = 1,45$$

$$\therefore \tan \left(x - \frac{\pi}{12} \right) = 0,725$$

$$\therefore x - \frac{\pi}{12} = 0,627\dots + \pi k, \quad k \in \mathbb{Z}$$

$$\therefore x \approx 0,889 + \pi k$$



$$2. \text{ (a) } 3 \sin \left(2x + \frac{\pi}{6} \right) = 1,5$$

$$\therefore \sin \left(2x + \frac{\pi}{6} \right) = \frac{1}{2}$$

$$\therefore 2x + \frac{\pi}{6} = \frac{\pi}{6} + 2\pi k, \quad k \in \mathbb{Z}$$

$$\therefore x = \pi k$$

$$\text{or } 2x + \frac{\pi}{6} = \pi - \frac{\pi}{6} + 2\pi k$$

$$\therefore 2x = \frac{2\pi}{3} + 2\pi k$$

$$\therefore x = \frac{\pi}{3} + \pi k$$

$$\text{(b) } x \in \left\{ -\pi; 0; \pi; 2\pi; -\frac{2\pi}{3}; \frac{\pi}{3}; \frac{4\pi}{3} \right\}$$

$$3. \quad \cos \left(3x - \frac{\pi}{36} \right) = -\cos \left(x + \frac{\pi}{36} \right)$$

$$\therefore \cos \left(3x - \frac{\pi}{36} \right) = \cos \left(\pi - \left(x + \frac{\pi}{36} \right) \right)$$

$$\therefore 3x - \frac{\pi}{36} = \pm \left(\frac{35\pi}{36} - x \right) + 2\pi k, \quad k \in \mathbb{Z}$$

$$\therefore 3x - \frac{\pi}{36} = \left(\frac{35\pi}{36} - x \right) + 2\pi k$$

$$\therefore 4x = \pi + 2\pi k$$

$$\therefore x = \frac{\pi}{4} + \frac{\pi}{2} k$$

$$\text{or } 3x - \frac{\pi}{36} = -\left(\frac{35\pi}{36} - x \right) + 2\pi k$$

$$\therefore 2x = -\frac{34\pi}{36} + 2\pi k$$

$$\therefore x = -\frac{17\pi}{36} + \pi k, \quad k \in \mathbb{Z}$$

$$4. \text{ (a) } \sin \left(2x + \frac{\pi}{6} \right) = \sin x$$

$$\therefore 2x + \frac{\pi}{6} = x + 2\pi k, \quad k \in \mathbb{Z}$$

$$\therefore x = -\frac{\pi}{6} + 2\pi k$$

$$\text{or } 2x + \frac{\pi}{6} = \pi - x + 2\pi k$$

$$\therefore 3x = \frac{5\pi}{6} + 2\pi k$$

$$\therefore x = \frac{5\pi}{18} + \frac{2\pi}{3} k$$

$$\text{(b) } x \in \left\{ -\frac{\pi}{6}; -\frac{7\pi}{18}; \frac{5\pi}{18}; \frac{17\pi}{18} \right\}$$

$$5. \quad \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) = -\tan x$$

$$\therefore \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) = \tan(-x)$$

$$\therefore \frac{x}{2} + \frac{\pi}{4} = -x + \pi k, \quad k \in \mathbb{Z}$$

$$\therefore \frac{3x}{2} = -\frac{\pi}{4} + \pi k$$

$$\therefore x = -\frac{\pi}{6} + \frac{2\pi k}{3}$$

$$6. \text{ (a) } \cos 2x = \sin x$$

$$\therefore \cos 2x = \cos \left(\frac{\pi}{2} - x \right)$$

$$\therefore 2x = \pm \left(\frac{\pi}{2} - x \right) + 2\pi k, \quad k \in \mathbb{Z}$$

$$\therefore 2x = \left(\frac{\pi}{2} - x \right) + 2\pi k$$

$$\therefore 3x = \frac{\pi}{2} + 2\pi k$$

$$\therefore x = \frac{\pi}{6} + \frac{2\pi}{3} k$$

$$\text{or } 2x = -\left(\frac{\pi}{2} - x \right) + 2\pi k$$

$$\therefore x = -\frac{\pi}{2} + 2\pi k$$

$$\text{(b) } x \in \left\{ -\frac{11\pi}{6}; -\frac{7\pi}{6}; -\frac{\pi}{2}; \frac{\pi}{6}; \frac{5\pi}{6} \right\}$$



$$7. \quad 2 \cos 2x = 1,3$$

$$\therefore \cos 2x = 0,65$$

$$\therefore 2x = \pm 0,8632\dots + 2\pi k, \quad k \in \mathbb{Z}$$

$$\therefore x \approx \pm 0,432 + \pi k$$

(h) **RTP:**

$$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + \dots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}} \rightarrow \text{A}$$

Proof:

For $n = 1$: LHS = 1 RHS = 1

\therefore **A** is true for $n = 1$

Assume **A** is true for $n = k, k \in \mathbb{N}$

i.e. $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + \dots + k\left(\frac{1}{2}\right)^{k-1} = 4 - \frac{k+2}{2^{k-1}}$

For $n = k + 1$:

$$\begin{aligned} \text{LHS} &= 1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + \dots + k\left(\frac{1}{2}\right)^{k-1} \\ &\quad + (k+1)\left(\frac{1}{2}\right)^k \\ &= 4 - \frac{k+2}{2^{k-1}} + (k+1)\left(\frac{1}{2}\right)^k \\ &= 4 - \frac{2k+4-k-1}{2^k} \\ &= 4 - \frac{k+3}{2^k} \end{aligned}$$

$$\text{RHS} = 4 - \frac{k+1+2}{2^{k+1-1}} = 4 - \frac{k+3}{2^k} = \text{LHS}$$

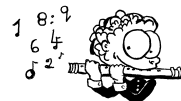
\therefore If **A** is true for $n = k$, then it is also true for $n = k + 1$.

A is true for $n = 1$.

\therefore **By Mathematical Induction, A is true for $n \in \mathbb{N}$.**

3. (a) **Assume the statement is true for $n = k, k \in \mathbb{N}$**

$$\begin{aligned} \text{(b) LHS} &= [1 + 8 + 27 + \dots + k^3] + [(k+1)^3] \\ &= \frac{k^2(k+1)^2}{4} + (k+1)^3 \\ &= \frac{k^2(k+1)^2 + 4(k+1)^3}{4} \\ &= \frac{(k+1)^2 [k^2 + 4k + 4]}{4} \\ &= \frac{(k+1)^2 (k+2)^2}{4} \\ &= \text{RHS} \end{aligned}$$



Exercise 13.2

(Questions – p. 143 in Book)

1. **RTP:** $n^2 + n$ is an even number

Proof:

For $n = 1$: $1^2 + 1 = 2$ which is an even number.

\therefore The statement is true for $n = 1$.

Assume that the statement is true for $n = k, k \in \mathbb{N}$

i.e. $k^2 + k = 2p, p \in \mathbb{N}$

For $n = k + 1$:

$$\begin{aligned} (k+1)^2 + (k+1) &= k^2 + 2k + 1 + k + 1 \\ &= (k^2 + k) + 2k + 2 \\ &= 2p + 2(k+1) \\ &= 2(p+k+1) \text{ which is divisible by 2.} \end{aligned}$$

\therefore If the statement is true for $n = k$, then it is also true for $n = k + 1$.

The statement is true for $n = 1$.

\therefore **by Mathematical Induction the statement is true for $n \in \mathbb{N}$.**

2. **RTP:** $n^3 + 2n$ is divisible by 3

Proof:

For $n = 1$: $1^3 + 2(1) = 3(1)$, which is divisible by 3
 \therefore The statement is true for $n = 1$

Assume the statement is true for $n = k, k \in \mathbb{N}$

i.e. $k^3 + 2k = 3p, p \in \mathbb{N}$

For $n = k + 1$:

$$\begin{aligned} (k+1)^3 + 2(k+1) &= k^3 + 3k^2 + 3k + 1 + 2k + 2 \\ &= (k^3 + 2k) + 3k^2 + 3k + 3 \\ &= 3p + 3(k^2 + k + 1) \\ &= 3(p + k^2 + k + 1) \text{ which is divisible by 3.} \end{aligned}$$

\therefore If the statement is true for $n = k$, then it is also true for $n = k + 1$.

The statement is true for $n = 1$.

\therefore **Statement is true for all $n \in \mathbb{N}, n \geq 1$ by the principle of Mathematical Induction.**

3. **RTP:**

$6n^2 + 2n$ is divisible by 4

Proof:

For $n = 1$: $6(1)^2 + 2(1) = 8 = 4(2)$, which is divisible by 4

\therefore The statement is true for $n = 1$.

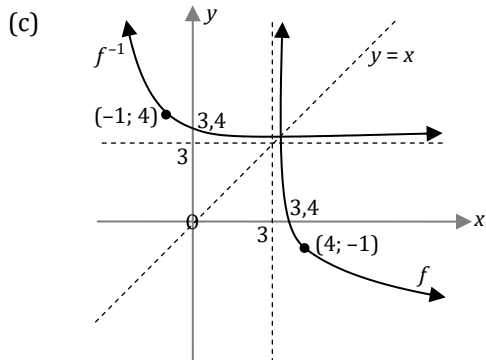
Assume the statement is true for $n = k, k \in \mathbb{N}$

i.e. $6k^2 + 2k = 4p, p \in \mathbb{N}$

For $n = k + 1$:

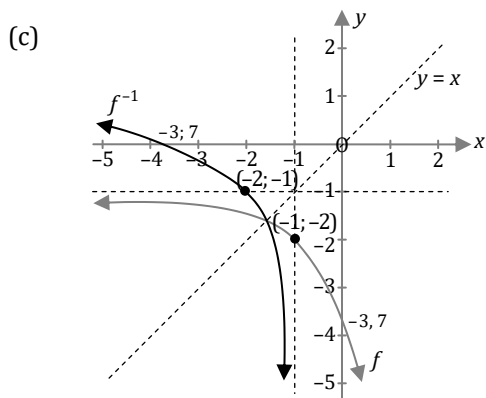
$$\begin{aligned} 6(k+1)^2 + 2(k+1) &= 6k^2 + 12k + 6 + 2k + 2 \\ &= (6k^2 + 2k) + 12k + 8 \\ &= 4p + 4(3k + 2) \\ &= 4(p + 3k + 2) \text{ which is divisible by 4.} \end{aligned}$$

Chapter 14: Exercise 14.7



2. (a) $f(x) = -e^{x+1} - 1$
 $\therefore x = -e^{y+1} - 1$
 $\therefore x + 1 = -e^{y+1}$
 $\therefore -x - 1 = e^{y+1}$
 $\therefore y + 1 = \ln(-x - 1)$
 $\therefore f^{-1}(x) = \ln(-x - 1) - 1$

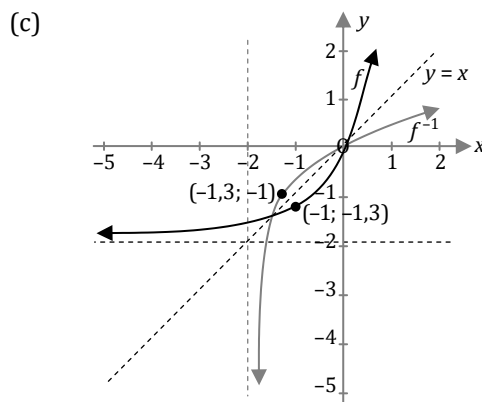
(b) $f(x)$
 Domain: $x \in \mathbb{R}$ Range: $y < -1$
 $f^{-1}(x)$
 Domain: $x < -1$ Range: $y \in \mathbb{R}$



3. (a) $f(x) = 2e^x - 2$
 $\therefore x = 2e^y - 2$
 $\therefore x + 2 = 2e^y$
 $\therefore \frac{x}{2} + 1 = e^y$
 $\therefore y = \ln\left(\frac{x}{2} + 1\right)$
 $\therefore f^{-1}(x) = \ln\left(\frac{x}{2} + 1\right)$

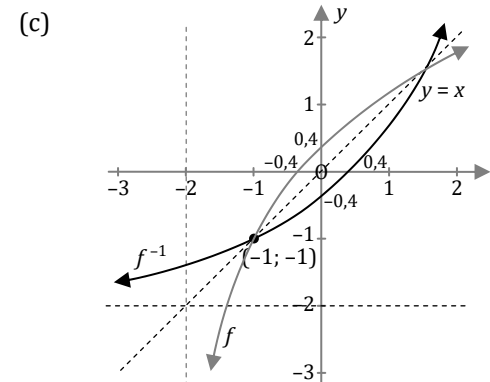


(b) $f(x)$
 Domain: $x \in \mathbb{R}$ Range: $y > -2$
 $f^{-1}(x)$
 Domain: $x > -2$ Range: $y \in \mathbb{R}$



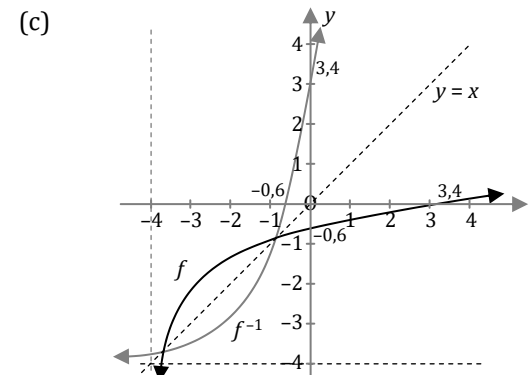
4. (a) $f(x) = 2 \ln(x + 2) - 1$
 $\therefore x = 2 \ln(y + 2) - 1$
 $\therefore x + 1 = 2 \ln(y + 2)$
 $\therefore \frac{x + 1}{2} = \ln(y + 2)$
 $\therefore y + 2 = e^{\frac{x+1}{2}}$
 $\therefore f^{-1}(x) = e^{\frac{x+1}{2}} - 2$

(b) $f(x)$
 Domain: $x > -2$ Range: $y \in \mathbb{R}$
 $f^{-1}(x)$
 Domain: $x \in \mathbb{R}$ Range: $y > -2$



5. (a) $f(x) = \ln(x + 4) - 2$
 $\therefore x = \ln(y + 4) - 2$
 $\therefore x + 2 = \ln(y + 4)$
 $\therefore y + 4 = e^{x+2}$
 $\therefore f^{-1}(x) = e^{x+2} - 4$

(b) $f(x)$
 Domain: $x > -4$ Range: $y \in \mathbb{R}$
 $f^{-1}(x)$
 Domain: $x \in \mathbb{R}$ Range: $y > -4$



$$(b) \quad y - 0 = -\frac{5}{2}(x - 8)$$

$$\therefore y = -\frac{5}{2}x + 20$$

2. $D_x(3y^4 + 4x - x^2 \sin y - 4) = D_x(0)$

$$\therefore 12y^3 \frac{dy}{dx} + 4 - (2x \sin y + x^2 \cos y \frac{dy}{dx}) = 0$$

$$\therefore 12y^3 \frac{dy}{dx} - x^2 \cos y \frac{dy}{dx} = 2x \sin y - 4$$

$$\therefore \frac{dy}{dx} = \frac{2x \sin y - 4}{12y^3 - x^2 \cos y}$$

At (1; 0) $m_T = \frac{2(1) \sin 0 - 4}{12(0)^3 - (1)^2 \cos 0} = 4$

$$\therefore m_N = -\frac{1}{4}$$

$$y - 0 = -\frac{1}{4}(x - 1)$$

$$\therefore y = -\frac{1}{4}x + \frac{1}{4}$$

3. (a) LHS = $1^3 + 2^3 - (1)(2^2) = 5$
RHS = $5 = \text{LHS}$

Point lies on curve.

(b) $D_x(x^3 + y^3 - xy^2) = D_x(5)$

$$\therefore 3x^2 + 3y^2 \frac{dy}{dx} - (y^2 + 2xy \frac{dy}{dx}) = 0$$

$$\therefore 3y^2 \frac{dy}{dx} - 2xy \frac{dy}{dx} = y^2 - 3x^2$$

$$\therefore \frac{dy}{dx} = \frac{y^2 - 3x^2}{3y^2 - 2xy}$$

(c) At (1; 2): $m_T = \frac{2^2 - 3(1)^2}{3(2)^2 - 2(1)(2)} = \frac{1}{8}; \therefore m_N = -8$

$$y - 2 = -8(x - 1)$$

$$\therefore y = -8x + 10$$

4. (a) $D_x(\sin y) = D_x(x)$

$$\therefore \cos y \times \frac{dy}{dx} = 1$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos y}$$

(b) $\sin y = 0,5$

$$\therefore y = \frac{\pi}{6} \left(0,5; \frac{\pi}{6}\right)$$

$$m_T = \frac{1}{\cos \frac{\pi}{6}} = \frac{2\sqrt{3}}{3}; \therefore m_N = -\frac{\sqrt{3}}{2}$$

$$y - \frac{\pi}{6} = -\frac{\sqrt{3}}{2}(x - 0,5)$$

$$\therefore y = -\frac{\sqrt{3}}{2}x + 0,96$$

5. $f(x) = e^{4x^2}$

$$f'(x) = e^{4x^2} \times 8x$$

$$m_T = f'\left(\frac{1}{4}\right) = e^{4\left(\frac{1}{4}\right)^2} \times 8\left(\frac{1}{4}\right) = 2,568\dots;$$

$$m_N \approx -0,39$$

$$f\left(\frac{1}{4}\right) = e^{4\left(\frac{1}{4}\right)^2} \approx 1,28$$

$$y - 1,28 = -0,39\left(x - \frac{1}{4}\right)$$

$$\therefore y = -0,39x + 1,38$$

6. $y = e^x; y' = e^x$

At $x = 0: m = 1$

$$y = e^{-x}; y' = -e^x$$

At $x = 0: m = -1$

$$m_T \times m_N = -1$$

\therefore **Graphs are orthogonal**



Gr 12 Exam Solutions (Questions - p. 176 in Book)

1. (a) $f(x) = \frac{1}{1 - 2x}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \left(\frac{1}{1 - 2(x+h)} - \frac{1}{1 - 2x} \right) \div h$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \cdot \frac{1 - 2x - (1 - 2x - 2h)}{(1 - 2x - 2h)(1 - 2x)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \cdot \frac{2h}{(1 - 2x - 2h)(1 - 2x)}$$

$$= \frac{2}{(1 - 2x)^2}$$

(b) $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \left(\frac{1}{\sqrt{2x+2h+1}} - \frac{1}{\sqrt{2x+1}} \right) \times \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2x+1} - \sqrt{2x+2h+1})(\sqrt{2x+1} + \sqrt{2x+2h+1})}{h\sqrt{2x+1}\sqrt{2x+2h+1}(\sqrt{2x+1} + \sqrt{2x+2h+1})}$$

$$= \lim_{h \rightarrow 0} \frac{2x+1 - (2x+2h+1)}{h\sqrt{2x+1}\sqrt{2x+2h+1}(\sqrt{2x+1} + \sqrt{2x+2h+1})}$$

$$= \lim_{h \rightarrow 0} \frac{-2h}{h\sqrt{2x+1}\sqrt{2x+2h+1}(\sqrt{2x+1} + \sqrt{2x+2h+1})}$$

$$= \frac{-2}{(2x+1)(2\sqrt{2x+1})}$$

$$= \frac{-1}{(2x+1)^{\frac{3}{2}}}$$

7. $f(x) = \frac{2x^2 - 5x + 1}{x - 1}$

Vertical: $x = 1$

Oblique: $f(x) = \frac{2x(x-1) - 3x + 1}{x-1}$
 $= \frac{2x(x-1) - 3(x-1) - 2}{x-1}$
 $= (2x-3) - \frac{2}{x-1}$

∴ O.A.: $y = 2x - 3$

8. Vertical: $2x^2 + 9x - 5 = (2x - 1)(x + 5) = 0$

∴ V.A.: $x = \frac{1}{2}; x = -5$

Horizontal: $\lim_{x \rightarrow \infty} \frac{x^2 \left(\frac{1}{x} - \frac{1}{x^2} \right)}{x^2 \left(2 + \frac{9}{x} - \frac{5}{x^2} \right)} = 0$

∴ H.A.: $y = 0$

9. $f(x) = \frac{3x - 5}{5 - 2x}$

Vertical: $x = \frac{5}{2}$

Horizontal: $\lim_{x \rightarrow \infty} \frac{x \left(3 - \frac{5}{x} \right)}{x \left(-2 + \frac{5}{x} \right)} = -\frac{3}{2}$

H.A.: $y = -\frac{3}{2}$



10. $f(x) = \frac{x^2 + 5x + 1}{x^3 - 7x^2 + 10x}$

Vertical: $x(x^2 - 7x + 10) = x(x-5)(x-2) = 0$

V.A.: $x = 0; x = 2; x = 5$

Horizontal: $\lim_{x \rightarrow \infty} \frac{x^3 \left(\frac{1}{x} + \frac{5}{x^2} + \frac{1}{x^3} \right)}{x^3 \left(1 - \frac{7}{x} + \frac{10}{x^2} \right)} = 0$

∴ H.A.: $y = 0$

Exercise 17.9

(Questions - p. 205 in Book)

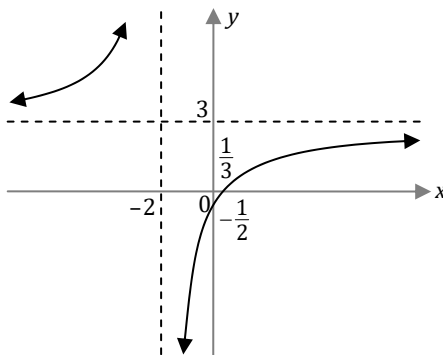
1. (a) $f(x) = \frac{3x - 1}{x + 2}$

y-intercept: $\left(0; -\frac{1}{2}\right)$ x-intercept(s): $\left(\frac{1}{3}; 0\right)$

$f(x) = \frac{3x - 1}{x + 2} = \frac{3(x + 2) - 7}{(x + 2)} = 3 - \frac{7}{(x + 2)}$

V.A.: $x = -2$ H.A.: $y = 3$

x	$x < -2$	$x = -2$	$-2 < x < \frac{1}{3}$	$x = \frac{1}{3}$	$x > \frac{1}{3}$
$f(x)$	+	u/d	-	0	+



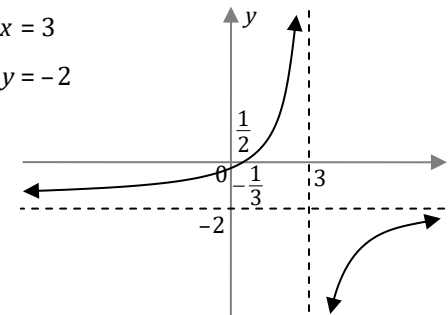
(b) $f(x) = \frac{1 - 2x}{x - 3}$

Intercepts: y-int.: $\left(0; -\frac{1}{3}\right)$ x-int.: $\left(\frac{1}{2}; 0\right)$

$f(x) = \frac{-2(x-3) - 5}{x-3} = -2 - \frac{5}{x-3}$

V.A.: $x = 3$

H.A.: $y = -2$



(c) $f(x) = \frac{x^2 - 4}{x + 1} = \frac{(x-2)(x+2)}{(x+1)}$

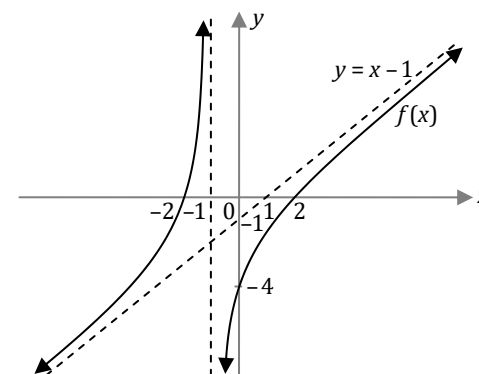
y-int.: $(0; -4)$ x-int.: $(-2; 0); (2; 0)$

$f(x) = \frac{x(x+1) - x - 4}{x+1} = \frac{x(x+1) - (x+1) - 3}{x+1}$
 $= (x-1) - \frac{3}{(x+1)}$

V.A.: $x = -1$

O.A.: $y = x - 1$

x	$x < -2$	$x = -2$	$-2 < x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$f(x)$	-	0	+	u/d	-	0	+



Chapter 18: Exercise 18.11 & 18.12

$$\begin{aligned}
 3. \quad & \int 2x(x+2)^9 dx \\
 & u = x+2 \quad x = u-2 \quad du = dx \\
 & \int 2(u-2)u^9 du \\
 & = 2 \int (u^{10} - 2u^9) du \\
 & = 2 \left(\frac{u^{11}}{11} - \frac{2u^{10}}{10} \right) + c \\
 & = 2u^{10} \left(\frac{u}{11} - \frac{1}{5} \right) + c \\
 & = 2u^{10} \cdot \frac{5u-11}{55} + c \\
 & = \frac{2(x+2)^{10}(5x+10-11)}{55} + c \\
 & = \frac{2(x+2)^{10}(5x-1)}{55} + c
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \int 9x(3x+2)^3 dx \\
 & u = 3x+2 \quad x = \frac{u-2}{3} \quad du = 3dx \\
 & \int 9 \cdot \frac{u-2}{3} \cdot u^3 \frac{du}{3} = \int (u^4 - 2u^3) du \\
 & = \frac{u^5}{5} - \frac{2u^4}{4} + c \\
 & = u^4 \left(\frac{u}{5} - \frac{1}{2} \right) + c \\
 & = u^4 \cdot \frac{2u-5}{10} + c \\
 & = \frac{(3x+2)^4(2(3x+2)-5)}{10} + c \\
 & = \frac{(3x+2)^4(6x-1)}{10} + c
 \end{aligned}$$



$$\begin{aligned}
 5. \quad & \int \frac{3x}{\sqrt{2x+3}} dx \\
 & u = 2x+3 \quad x = \frac{u-3}{2} \quad du = 2dx \\
 & \int 3 \cdot \frac{u-3}{2} \cdot u^{-\frac{1}{2}} \cdot \frac{du}{2} = \frac{3}{4} \int \left(\frac{1}{2} u^{\frac{1}{2}} - 3u^{-\frac{1}{2}} \right) du \\
 & = \frac{3}{4} \left(\frac{\frac{3}{2} u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3 \cdot 2u^{\frac{1}{2}}}{1} \right) + c \\
 & = \frac{3u^{\frac{3}{2}}}{2} \left(\frac{u}{3} - 3 \right) + c \\
 & = \frac{3u^{\frac{3}{2}}}{2} \cdot \frac{u-9}{3} + c \\
 & = \frac{\sqrt{2x+3}(2x+3-9)}{2} + c \\
 & = \sqrt{2x+3}(x-3) + c
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \int_1^2 x^2(x-1)^5 dx \\
 & u = x-1 \quad x = u+1 \\
 & du = dx \quad x^2 = (u+1)^2 = u^2 + 2u + 1 \\
 & x = 1, u = 0 \quad x = 2, u = 1 \\
 & \int_0^1 (u^2 + 2u + 1)u^5 du = \int_0^1 (u^7 + 2u^6 + u^5) du \\
 & = \left[\frac{u^8}{8} + \frac{2u^7}{7} + \frac{u^6}{6} \right]_0^1 \\
 & = \left(\frac{1}{8} + \frac{2}{7} + \frac{1}{6} \right) - 0 \\
 & = \frac{97}{168}
 \end{aligned}$$

Exercise 18.12

(Questions – p. 229 in Book)

$$\begin{aligned}
 1. \quad & \int \frac{x-1}{x+1} dx \\
 & \frac{x-1}{x+1} = \frac{x+1-2}{x+1} = 1 - \frac{2}{x+1} \\
 & \int \left(1 - \frac{2}{x+1} \right) dx = x - 2\ln|x+1| + c
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \int \frac{x+3}{x-5} dx \\
 & \frac{x+3}{x-5} = \frac{x-5+8}{x-5} = 1 + \frac{8}{x-5} \\
 & \int \left(1 + \frac{8}{x-5} \right) dx = x + 8\ln|x-5| + c
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \int \frac{x^2 - 2x + 3}{x} dx \\
 & \frac{x^2 - 2x + 3}{x} = x - 2 + \frac{3}{x} \\
 & \int \left(x - 2 + \frac{3}{x} \right) dx = \frac{x^2}{2} - 2x + 3\ln|x| + c
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \int \frac{x^3}{x^2-4} dx \\
 & \frac{x^3}{x^2-4} = \frac{x(x^2-4)+4x}{x^2-4} = x + \frac{4x}{x^2-4} \\
 & \int \left(x + \frac{4x}{x^2-4} \right) dx = \int x dx + \int \frac{4x}{x^2-4} dx \\
 & = \frac{x^2}{2} + 4 \int \frac{2x}{2} \cdot \frac{1}{x^2-4} dx \\
 & = \frac{x^2}{2} + 2\ln|x^2-4| + c
 \end{aligned}$$