Important advice for matrics – the final stretch

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We trust that working through these exam papers and following our suggested answers and comments will help you prepare thoroughly for your final exam.

The Answer Series Maths study guides offer a key to exam success.
IMPORTANT ADVICE FOR MATRICS – THE FINAL STRETCH!

Take a deep breath!

Don't focus on what you haven't done in the past. Put that behind you and start today! Give it your all – it is well worth it!

TIMETABLE

1. Draw up a timetable of study days and times. As soon as you receive your actual exam timetable, revise your schedule to ensure focussed preparation and awareness of time.

ROUTINE

1. Routine is really important. Start early in the morning, at the same time every day, and don't work beyond 11 at night.

   - Arrange some 1 hour and some 2 hour sessions.
   - Schedule more difficult pieces of work for early in the day and easier bits for later when you're tired.
   - Reward yourself with an early night now and again!
   - Allow some time for physical exercise – at least ½ hour a day. Walking, jogging (or skipping when it rains) will improve your concentration.

'NO-NO'S'

1. Stay away from
   - your phone (maximum ½ hour)
   - the television (maximum ½ hour)
   - Facebook and any other social networks – NONE!
   - the sun – NONE!

   All these break down your commitment, focus and energy.

PLANNING

1. Spend a day just planning the work for your subjects. Take each one and write down what needs to be done for each.

   - Allocate specific pieces of work to each session on your timetable.

   Motivation will not be a problem once you've done this, because you will see that you need to use every minute!

COMMUNICATE

1. Tell your family about your timetable. Paste it on your door, so that no one disturbs you. Your parents won't nag you once they see you taking responsibility and doing your best.

WORK FOCUS

1. Don't worry about marks. Just focus on the work and the marks will take care of themselves. Worrying is tiring and time-wasting and gets in the way of doing your work! Your marks will be okay if you work hard.

EXAM PREPARATION

While working past papers is excellent preparation for any exam – and The Answer Series provides these – WORKING ON ONE TOPIC AT A TIME is most effective, particularly as you build up your confidence. The Answer Series provides thorough topic treatment for all subjects.

A GREAT STUDY TIP

Don't just read through work! Study a section and then, on your own, write down all you can remember. Knowing that you're going to do this makes you study in a logical, alert way. You're then only left to learn the few things which you left out. This applies to all subjects.

THE EXAMS

Finally, for the exams themselves, make sure you have all you need, don't arrive too early and allow yourself to be upset by panicking friends. Plan your time in the exam well – allowing some time to check at the end. Whatever you do, don't allow yourself to get stuck on any difficult issues in the exam. Move on, and rather come back to problem questions if you have time left. If you're finding an exam difficult, just continue to do your absolute best right until the end!

YOUR APPROACH

The most important thing of all is to remain positive throughout until the entire exam session is over. Some times will be tough, some exams WILL BE TOUGH, but in the end, your results will reflect all the effort that you have put in.

ABOUT PREPARING FOR THE MATHS EXAM

1. Try each problem on your own first – no matter how inadequately – before consulting the solutions. It is only by encountering the difficulties which you personally have that you will be able, firstly, to pin-point them, and then, secondly, to rectify them (and be receptive to learning) (i.e. “Make your mistakes, see what they are and then make sure you don’t make them again!!)

2. Learn to keep asking yourself “why”? It is when you learn to REASON that you really start enjoying maths and, quite coincidentally, start doing well at it!!

   **Answers are by no means the most important thing in mathematics.** When you've done a problem, don't be satisfied only to check the answer. Check also on your **layout** and reasoning (logic). Systematic, to-the-point, logical and neat presentation is very important.

3. Despair can destroy your mathematics. Mathematics should be taken on as a continual challenge (or not at all!). Teach your ego to suffer the "knocks" which it may receive – like a poor test result. Instead of being negative about your mistakes (e.g. "I'll never be able to do these sums"), **learn** from them and let them help you to understand.


We wish you the best of luck in the busy time which lies ahead and hope that this book will be the key to your success – enjoy it!!

Anne Eadie and Gretel Lampe
SECTION A

QUESTION 1

1.1 Solve for \( x \):

1.1.1 \((x + 2)^2 = 3x(x - 2)\)

giving your answers correct to one decimal digit. (4)

1.1.2 \(x^2 - 9x \geq 36\) (4)

1.1.3 \(3x - 3x^{-2} = 72\) (4)

1.2 Given: \((2m - 3)(n + 5) = 0\)

Solve for:

1.2.1 \(n\) if \(m = 1\) (1)

1.2.2 \(m\) if \(n \neq -5\) (1)

1.2.3 \(m\) if \(n = -5\) (2)

1.3 Solve for \( x \):

\[ (\sqrt{x - 1} - 3)(\sqrt{x - 1} + 2) = 0 \] (3) [19]

QUESTION 2

2.1 Evaluate:

\[ \sum_{k=2}^{6} \frac{2k-1}{k} \] (3)

2.2 The number of members of a new social networking site doubles every day. On day 1 there were 27 members and on day 2 there were 54 members.

2.2.1 Calculate the number of members there were on day 12. (2)

2.2.2 The site earns half a cent per member per day. Calculate the amount of money that the site earned in the first 12 days. Give your answer to the nearest Rand. (4)

2.3 Gina plans to start a fitness programme by going for a run each Sunday. On the first Sunday she runs 1 km and plans to increase the distance by 750 m each Sunday. When Gina reaches 10 km, she will continue to run 10 km each Sunday thereafter.

2.3.1 Calculate the distance that Gina will run on the 9th Sunday. (3)

2.3.2 Determine on which Sunday Gina will first run 10 km. (2)

2.3.3 Calculate the total distance that Gina will run over the first 24 Sundays. (4) [18]

QUESTION 3

3.1 Given: \(f(x) = 6x^2\), determine \(f'(x)\) from first principles. (4)

3.2 Determine \(f'(x)\) given \(f(x) = \frac{3x^4 + 7x^2 - 5x}{2x^2}\).

Leave your answer with positive exponents. (3)

3.3 Given: \(f(x) = x^3 - 7x^2 + 7x + 15\)

Determine the average gradient of the curve between the points where \(x = -1\) and \(x = 1\). (3) [10]

QUESTION 4

4.1.1 Joe invested a sum of R50 000 in a bank.

The investment remained in the bank for 15 years, earning interest at a rate of 6\% p.a., compounded annually.

Calculate the amount at the end of 15 years. (2)

4.1.2 Financial gain is defined as the difference between the final value of an investment and the contribution. Determine the financial gain of Joe's investment. (1)

4.2 Pumla took a mortgage loan of R850 000 to buy a house and was required to pay equal monthly instalments for 30 years. She was charged interest at 8\% p.a., compounded monthly.

4.2.1 Show that her monthly instalment was R6 237. (4)

4.2.2 Calculate the outstanding balance on her loan at the end of the first year. (3)

4.2.3 Hence calculate how much of the R74 844 that she paid during the first year, was taken by the finance company as payment towards the interest it charged. (3) [13]

SECTION B

QUESTION 5

5.1 \(\sqrt[12]{8}\) is a special number in music.

On an idealised piano, the frequency \(f(n)\) of the \(n^{th}\) key, in Hertz, is given by

\[ f(n) = \left(\frac{12}{13}\right)^{n-49} \times 440. \]

5.1.1 Calculate the frequency of the 73^{rd} key. (2)

5.1.2 Determine which key has a key frequency of 3 520 Hz. (4)

We trust that working through these exam papers and following our detailed answers and comments will help you prepare thoroughly for your final exam.

The Answer Series study guides offer a key to exam success in several major subjects.

In particular, Gr 12 Maths 2 in 1 offers 'spot-on' exam practice in separate topics and on CAPS-constructed Maths exam papers.

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Gr 12 Maths CAPS-constructed Exam: Paper 1

Q 1

5.2 Refer to the figure showing the graphs of \( f(x) = 3x - 1 \) and \( g(x) = x^2 \) intersecting at A(1; 2) and B(3; 8). C(2; 1; 4; 3) is a point on g, coordinates rounded to one decimal digit, such that the tangent to g at C is parallel to f.

5.2.1 Determine the equations of \( y = f^{-1}(x) \) and \( y = g^{-1}(x) \). (4)

5.2.2 Use the above graphs to determine the values of \( x \) for each of the following:

(i) \( f(x) < g(x) \) (2)

(ii) \( g^{-1}(x) < 0 \) (2)

(iii) \( f^{-1}(x) = g^{-1}(x) \) (2)

(iv) \( g'(x) > f'(x) \) (2) [18]

Note: Be careful to note the difference between \( f^{-1} \), the inverse function, and \( f' \), the derivative.

QUESTION 6

Refer to the figure showing the graph of \( f(x) = x^2 \).

A and B are any two different points on the parabola.
The tangents at A and B intersect at C.

Given the \( x \)-coordinate of A is \( k \) and the \( x \)-coordinate of B is \( m \).

6.1 Show that the equation of the tangent at A can be written as \( y = 2kx - k^2 \). (5)

6.2 Hence, write down the equation of the tangent at B. (1)

6.3 Determine a simplified expression for the \( x \)-coordinate of C. (4)

6.4 D is the midpoint of the line segment between A and B. Show that CD is parallel to the y-axis. (2) [12]

QUESTION 7

Refer to the figure showing the graph of a cubic function:

\( f(x) = ax^3 + bx^2 + cx + d \)

A(-6; 0), B(-1; 0), C(2; 0) and F(0; 24) are intercepts with the axes, with D and E as turning points.

7.1 Show that \( a = -2 \), \( b = -10 \), \( c = 16 \) and \( d = 24 \). (5)

7.2 Determine the coordinates of D. (5)

7.3 Suppose that the graph is translated in such a way that the point D is moved to the origin. That is, the new graph has equation \( y = f(x - p) + q \), where \( p \) and \( q \) are constants.

Write down the values of \( p \) and \( q \). (2) [12]

QUESTION 8

Refer to the figure showing the parabola given by

\( f(x) = 4 - \frac{x^2}{4} \) with \( 0 \leq x \leq 4 \).

D is the point \((x; 0)\) and DB is parallel to the y-axis, with B on the graph of \( f \).

8.1 Write down the coordinates of B in terms of \( x \). (2)

8.2 Show that the area, A, of \( \triangle OBD \) is given by:

\[ A = 2x - \frac{x^3}{8} \]. (3)

8.3 Determine how far D should be from O in order that the area of \( \triangle OBD \) is as large as possible. (5)

8.4 Hence, calculate the area of \( \triangle OBD \) when D is the point determined in Question 8.3. (2) [12]

QUESTION 9

Refer to the figure showing the graph of \( f(x) = -x^2 + 4x \) followed by a number of decreasing sized parabolas.
The height of each turning point as well as the difference between the \( x \)-intercepts of each parabola is \( \frac{3}{4} \) of that of the previous parabola.

9.1 Determine the coordinates of A and E. (6)

9.2 Show that the coordinates of G are \( \left(\frac{65}{8}, \frac{9}{4}\right) \). (6)

9.3 Determine the equation of the third parabola passing through B, G and C, leaving your answer in the form \( y = a(x - p)^2 + q \). (4)

9.4 Suppose that decreasing parabolas are constructed indefinitely in the same way as the first few that are shown. Determine whether all the parabolas will fit on OH, where \( OH = 15 \). (3) [19]

Note: Be careful to note the difference between \( f^{-1} \), the inverse function, and \( f' \), the derivative.
QUESTION 10
10.1 If A and B are independent events, find the values of x and y. All working must be shown.

10.2 The table summarises the results of all the language tests taken at a Language Centre in Cape Town during the first week of January.

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass</td>
<td>32</td>
<td>43</td>
<td>75</td>
</tr>
<tr>
<td>Fail</td>
<td>8</td>
<td>15</td>
<td>23</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>58</td>
<td>98</td>
</tr>
</tbody>
</table>

A person is chosen at random from those who took their test during the first week of January.

10.2.1 Find the probability that the person was a male who failed. (2)

10.2.2 The person chosen is a female. Find the probability that she passed the test. (2)

QUESTION 11
All answers containing factorials must be calculated e.g.: 4! = 24

11.1 In how many ways can the letters of the word Geometry be arranged, if the letter G is at the beginning? (3)

11.2 Three men (Piet, Jabu and John) and 2 women (Sipho and Jane) are to stand in a straight line to have their group photograph taken.

Find the probability that Piet stands next to Sipho and Jabu stands next to Jane. (5)

TOTAL: 150

QUESTION 1
The following table gives the frequency distribution of the daily travelling time (in minutes) from home to work for the employees of a certain company.

<table>
<thead>
<tr>
<th>Daily travelling time (in minutes)</th>
<th>Number of employees</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ x &lt; 10</td>
<td>20</td>
</tr>
<tr>
<td>10 ≤ x &lt; 20</td>
<td>35</td>
</tr>
<tr>
<td>20 ≤ x &lt; 30</td>
<td>30</td>
</tr>
<tr>
<td>30 ≤ x &lt; 40</td>
<td>10</td>
</tr>
<tr>
<td>40 ≤ x &lt; 50</td>
<td>5</td>
</tr>
</tbody>
</table>

1.1 Circle the correct answer for the following questions:

1.1.1 The estimated mean time (in minutes) taken by employees is:
A 14,5  B 19,5  C 16,7  D 24,5 (2)

1.1.2 The estimated standard deviation for the time (in minutes) is:
A 10,57  B 14,14  C 114,75  D 10,71 (2)

1.2 An ogive was constructed from the given data.
Construct a box-and-whisker plot on the scaled axis below the ogive, to summarise the given data. (3)

1.3 State whether the following is TRUE or FALSE.

1.3.1 The distribution of these travelling times is skewed positively.
1.3.2 The inter-quartile range for this data is 25.
1.3.3 Only 35 employees take less than 20 minutes. (3)

QUESTION 2
Mr Ryan is a retired teacher who supplements his pension by mowing lawns for customers who live in his neighbourhood.

As part of a review of his charges for this work, he measures the approximate areas (x) (in m²) of a random sample of 12 of his customers’ lawns and notes the time (y) in minutes, that it takes him to mow these lawns.

His results are shown in the table.

<table>
<thead>
<tr>
<th>Area (x) (m²)</th>
<th>Time (y) (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>360</td>
<td>50</td>
</tr>
<tr>
<td>120</td>
<td>25</td>
</tr>
<tr>
<td>645</td>
<td>120</td>
</tr>
<tr>
<td>602</td>
<td>130</td>
</tr>
<tr>
<td>1190</td>
<td>75</td>
</tr>
<tr>
<td>530</td>
<td>120</td>
</tr>
<tr>
<td>245</td>
<td>95</td>
</tr>
<tr>
<td>486</td>
<td>55</td>
</tr>
<tr>
<td>1350</td>
<td>70</td>
</tr>
<tr>
<td>350</td>
<td>48</td>
</tr>
<tr>
<td>1100</td>
<td>110</td>
</tr>
<tr>
<td>320</td>
<td>55</td>
</tr>
<tr>
<td>250</td>
<td>60</td>
</tr>
</tbody>
</table>

2.1 Use your calculator to determine the equation of the least squares regression line. Give your answers correct to 4 decimal digits. (4)
2.2 Calculate the value of \( r \), the correlation coefficient for the data, correct to 4 decimal digits. (2)

2.3 Given that Mr Ryan charges a flat call out fee of R150, as well as R50 per half hour (or part thereof), estimate the charge for mowing a customer's lawn that has an area of 560 \( \text{m}^2 \).
(For example: 100 minutes would be taken as 2 hours) (3)

2.4 The local high school wants Mr Ryan to mow their rugby field which is rectangular, 100 metres long by 70 metres wide. Should you use the regression equation found in Question 2.1 to calculate the time it would take to mow this area? Give a reason for your answer. (2) [11]

QUESTION 3

3.1 In the diagram below, \( \triangle \text{SAO} \) has vertices S(12; 16), A(4; 16) and O(0; 0).
K is the midpoint of AS and AT is perpendicular to OS with T a point on OS.

**3.1.1** Determine the coordinates of K and hence the equation of line OK. (5)

**3.1.2** Determine the gradient of OS and hence the equation of line AT in the form \( y = mx + c \). (4)

**3.1.3** Determine, correct to one decimal digit, the size of:
(a) \( \hat{1}_1 \) (b) \( \hat{2}_2 \) (2)(3)
(c) Hence, or otherwise, determine the size of \( R_1 \) and \( K_1 \). (3)

3.2 If B(-8; 4) and D(4; -8), determine the equation of the circle having BD as a diameter. (4) [21]

**QUESTION 4**

4.1 In the diagram alongside, a circle has a diameter with equation \( y = 2x + 3 \).
The tangent at point E on the circle cuts the x-axis at F(12; 0). Determine the coordinates of E. (6)

4.2 In the diagram below, two circles are drawn. Circle O touches circle centre B externally.
The equation of the circle centre O is given by \( x^2 + y^2 = 45 \).
The equation of the circle centre B is given by \( (x - 2p)^2 + (y + p)^2 = 20 \).

**Determining the value of p.** (5)

**4.3** Prove that the radius of the circle having equation \( x^2 + y^2 + 4x \cos \theta + 8y \sin \theta + 3 = 0 \) can never exceed \( \sqrt{13} \) for any value of \( \theta \). (5) [16]

**QUESTION 5**

5.1 Given: \( \cos \hat{G} = 0.726 \) and \( 180^\circ < \hat{G} < 360^\circ \).

**5.1.1** Use a calculator to determine \( \hat{G} \), correct to one decimal digit. (2)

**5.1.2** Hence determine the value of \( \tan \left( \frac{2}{3} \hat{G} + 100^\circ \right) \), correct to three decimal digits. (1)

5.2 Simplify as far as possible:
\[
\frac{\sin(180^\circ - A)}{\cos(90^\circ + A) + \sin(360^\circ - A)}
\]

5.3 In the diagram alongside, T(8; k) is a point in the first quadrant.
If \( \tan \beta = \frac{1}{4} \), determine without using a calculator:

**5.3.1** the value of \( k \). (2)

**5.3.2** the value of \( \sin \beta \).
Leave your answer in simplified surd form. (3)

5.4 Simplify without the use of a calculator:
\[
\frac{\cos(45^\circ - \theta)}{\cos 45^\circ \cos \theta + \tan \theta}
\] (5) [17]

**QUESTION 6**

6.1 The graphs of \( y = \cos ax \) and \( y = \tan bx \) are sketched for \( a \in [0^\circ; 180^\circ] \).

**6.1.1** Write down the period of \( y = \cos ax \). (1)

**6.1.2** Write down the value of \( a \). (1)

**6.1.3** Write down the period of \( y = \tan bx \). (1)

**6.1.4** Write down the value of \( b \). (1)
6.2 The graphs of \( f(x) = 2 \sin x \), \( g(x) = 1.5 \cos x \) and \( h(x) = 4 \tan x \) are drawn for \( x \in [0^\circ; 60^\circ] \).

6.2.1 Determine the coordinates of \( P \) in simplest surd form. (2)

6.2.2 Determine the coordinates of \( Q \) correct to two decimal digits. (4) [10]

**QUESTION 7**
In \( \triangle PQR \) below, \( PQ = 2 \) and \( QR = 1 \).
\( S \) is the midpoint of \( PQ \).
\( P\hat{R}S = \alpha \) and \( R\hat{S}Q = \theta \).

7.1 Determine \( \hat{P} \) in terms of \( \theta \) and \( \alpha \). (1)

7.2 Show that \( \tan \theta = 3 \tan \alpha \). (6) [7]

**QUESTION 8**
In \( \triangle LMN \), \( LM = 5 \) units, \( LN = 2x \) units and \( MN = 3x \) units.

8.1 Prove that \( \cos L = \frac{5 - x^2}{4x} \). (4)

8.2 Give the restrictions for \( \cos L \) if \( \hat{L} \) is obtuse. (2)

8.3 Is it possible for \( x \) to be equal to 6? (1)

8.4 If \( x = 3 \), calculate the area of \( \triangle LMN \), rounded off to one decimal digit. (4) [11]

**QUESTION 9**
In each case below, you are given a statement and a reason that are true for the incomplete diagram. Complete the diagram, showing what was necessary so that the statement and the reason are true.

9.1 Statement:
\( \hat{A}OB = 2\hat{A}\hat{C}B \).

Reason:
\( \angle \) at centre equals \( 2 \times \angle \) at the circumference.

9.2 Statement:
\( TS = SP \).

Reason:
line from centre perpendicular to chord.

9.3 Statement:
\( \hat{B}\hat{A}\hat{D} = \hat{T} \).

Reason:
tan chord theorem.

**QUESTION 10**
From a point \( A \) outside the circle, centre \( O \), two tangents \( AD \) and \( AV \) are drawn. \( AO \) and \( VD \) meet in \( M \).
\( BOD \) is a diameter of the circle. \( BV \) and \( VO \) are drawn.

Let \( \hat{A}_1 + \hat{A}_2 = 40^\circ \).

10.1 Complete the following table:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1.1 ( \hat{D} = 90^\circ )</td>
<td>10.1.2 ( \hat{B} = 90^\circ )</td>
</tr>
</tbody>
</table>

10.2 Calculate, with reasons, the size of:
10.2.1 \( \hat{D}_1 \) \( (2) \)
10.2.2 \( \hat{O}_1 \) \( (2) \)

10.3 Prove, with reasons, that \( BV \parallel OA \), i.e. \( BV \parallel OA \). \( (3) [11] \)

**QUESTION 11**
In the diagram, which is not drawn to scale, \( KLJC \) is a trapezium with \( KL \parallel CJ \).

\( CK = 24 \) cm, \( KL = 8 \) cm, \( LJ = 12 \) cm, \( JC = 32 \) cm and \( KJ = 16 \) cm.

Determine the ratio: \( \frac{\text{Area of } \triangle KLJ}{\text{Area of } \triangle CKJ} \). \( [5] \)
QUESTION 12
In the figure below AE is a diameter of circle ANE.
L is a point on AN and LE bisects \( \hat{AEN} \).
Let \( \hat{E}_1 = \hat{E}_2 = x \).

NE produced meets a line from A parallel to LE, in D.
Hence \( LE \parallel AD \).

12.1 Complete the following table:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{E}_1 = \hat{D} )</td>
<td></td>
</tr>
<tr>
<td>( \hat{E}_2 = \hat{A}_2 )</td>
<td></td>
</tr>
<tr>
<td>( \therefore AE = ED )</td>
<td>(3)</td>
</tr>
</tbody>
</table>

12.2 If \( NE = 12 \) units and the diameter of the circle is
20 units, calculate giving reasons:
12.2.1 AN (3)
12.2.2 AL (4) [10]

QUESTION 13
Two circles intersect at D and E. Chord RE of the smaller
circle is a tangent to the larger circle at E. N is a point on
the small circle.
EN and RD are produced to meet the bigger circle at A.
RN, ED and DN are drawn.
V is a point on the larger circle and AV and EV are drawn.

13.1 Complete the following table:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{N}_1 = \hat{E}_2 )</td>
<td>(1)</td>
</tr>
<tr>
<td>( \hat{E}_2 = \hat{A}_2 )</td>
<td>(1)</td>
</tr>
<tr>
<td>( \therefore \hat{N}_1 = \hat{A}_2 )</td>
<td></td>
</tr>
</tbody>
</table>

13.2 Prove \( \hat{D}_1 = \hat{E}_1 + \hat{E}_2 \).
13.3 Prove, with reasons, that \( \triangle EDR \parallel \triangle AER \).
13.4 If \( 2AV = DR, AR \) and \( ER = 3 \text{ cm} \), find the
length of \( AV \).

QUESTION 14
In the figure \( B \hat{A}D = 90^\circ \) and \( AC \perp BD \).

14.1 Complete, without giving reasons, the following:
\[ \triangle ABD \parallel \triangle \ldots \ldots \parallel \triangle \ldots \ldots \] (2)

14.2.1 \( \frac{AB}{BC} = \frac{BD}{AB} \ldots \ldots (\triangle ABD \parallel \triangle \ldots \ldots) \)
\( \therefore AB^2 = \ldots \ldots \) (2)

14.2.2 \( \frac{AD}{CD} = \frac{BD}{AD} \ldots \ldots (\triangle ABD \parallel \triangle \ldots \ldots) \)
\( \therefore AD^2 = \ldots \ldots \) (2)

14.3 Now prove that \( AB^2 + AD^2 = BD^2 \). (2) [8]

TOTAL: 150
1.3 \((\sqrt{x} - 1)(\sqrt{x} - 2) = 0\)

\[
\Rightarrow \sqrt{x} - 1 = 3 \quad \text{or} \quad \sqrt{x} - 2 = 0
\]

\[
\therefore x = 10
\]

2.1 \(\sum\) means 'the SUM of the terms'.

\[
\frac{6}{k=2} \sum k = \frac{2}{2} + \frac{3}{3} + \frac{4}{4} + \frac{5}{5} + \frac{6}{6}
\]

\[
= 2 + 1 + 1 + 1 + 1 = 6
\]

\[
= 1 + \frac{4}{3} + 2 + 3 + \frac{5}{3} = 9
\]

\[
= 193 \div 15 = 12,9
\]

2.2 G.S.: 27 ; 54 ; 

In 2.2.1: we need to determine \(T_{12}\)

In 2.2.2: we need to determine \(S_{12}\)

NB: Distinguish between:

\(T_{12}\): the 12th term & \(S_{12}\): the sum of 12 terms

2.2.1 \(a = 27\) ; \(r = 2\) ; \(T_{12}\)? ; \(n = 12\)

\[
T_n = a \cdot r^{n-1} \Rightarrow T_{12} = 27 \cdot 2^{12-1} = 55296
\]

2.2.2 \(S_{12}\)? ; \(n = 12\) ; \(a = 27\) ; \(r = 2\)

\[
S_n = \frac{a(r^n - 1)}{r - 1} \Rightarrow S_{12} = \frac{27(2^{12} - 1)}{2 - 1} = 110565
\]

\[
= 66282.5 \text{ cents}
\]

\[
= R552,825
\]

\[
= R553 \quad \text{correct to the nearest rand}
\]

2.3 NB: Note the units: they must be the same.

So, convert 750 m to 0.75 km.

\[
1 ; 1.75 ; 2.5 ; \ldots ; 10 ; 10 ; 10 ; \ldots
\]

2.3.1 \(a = 1\) ; \(d = 0.75\) ; \(n = 9\) ; \(T_9\) . . . A.S.

\[
T_n = a + (n - 1)d \Rightarrow T_9 = 1 + (9 - 1)(0.75)
\]

\[
= 7
\]

\(\therefore 7\) km

2.3.2 \(n?\) ; \(T_n = 10\) ; \(a = 1\) ; \(d = 0.75\)

\[
a + (n - 1)d = 10
\]

\[
0.75n - 0.75 = 9
\]

\[
0.75n = 9.75
\]

\[
= n = 13
\]

\(\therefore\) The 13th Sunday

2.3.3 The total distance = \(S_{13} + (11 \times 10)\) km

\[
S_{13} = \frac{n}{2} [2a + (n - 1)d] \Rightarrow S_{13} = \frac{13}{2} [2(1) + (13 - 1)(0.75)] = 71.5
\]

\(\therefore\) The total distance = 181.5 km

Note: After the 13th week, the distance remained

10 km every Sunday. So, the last 11 terms are all 10 km.

\(\therefore\) The sum of the distances = \(S_{13} + (11 \times 10)\)

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3.3 \( f(x) = x^3 - 7x^2 + 7x + 15 \)
\[ f(-1) = (-1)^3 - 7(-1)^2 + 7(-1) + 15 = 0 \]
\[ f(1) = 1^3 - 7(1)^2 + 7(1) + 15 = 16 \]

**NB:** The average gradient does not involve the derivatives:
only \( f(1), \ f(-1) \) and \( x = 1 \) and \( x = -1. \)

Use \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

Average gradient = \( \frac{f(1) - f(-1)}{1 - (-1)} \)
\[ = \frac{16 - 0}{2} = 8 \]

4. **NB:** Whereas the general instruction for the paper is to round off to 1 decimal digit, this does not apply to money where we round off to 2 decimal digits, indicating cents.

4.1.1 \( P = \text{R} 50,000 \; ; \; n = 15 \; ; \; i = 6\% = \frac{6}{100} = 0.06 \; ; \; A? \)
\[ A = P(1 + i)^n \implies A = 50,000(1 + 0.06)^{15} \]
\[ = \text{R} 119,827.91 \]

4.1.2 The financial gain = \( \text{R} 119,827.91 - \text{R} 50,000 \)
\[ = \text{R} 69,827.91 \]

4.2.1 \( P_v = \text{R} 850,000 \; ; \; i = \frac{8\%}{12} = 0.08 \; ; \; n = 30 \times 12 = 360 \; ; \; x? \)

You could store \( 0.08 \) in \( A \) memory. It would be useful throughout Q 4.2.

**Method 1:** Using \( P_v = \frac{x(1 + i)^n}{i} \ldots \) the Present value formula
\[ \text{R} 850,000 = \frac{x(1 + 0.08)^{360}}{0.08} \]
\[ \therefore \text{R} 850,000 = x \times 136,283,494 \]
\[ \therefore x = \frac{\text{R} 850,000}{136,283,494} \]
\[ = 6,236,998 \ldots \approx \text{R} 6,237 \]

**Method 2:** Using the Future value formula
\( F_v = P_v(1 + i)^n \rightarrow F_v = \text{R} 850,000(1 + 0.08)^{360} \]
\[ = \text{R} 9,295,370,209 \ldots \text{store in} \ A \]
\[ & F_v = \frac{x(1 + i)^n - 1}{i} \implies F_v = \frac{x(1 + 0.08)^{360} - 1}{0.08} \]
\[ = \frac{x(1 + 0.08)^{360} - 1}{0.08} \]
\[ = x \times 1,490,359 \ldots \text{store in} \ A \]
\[ A = \frac{x}{B} \implies x = \frac{A}{B} \approx \text{R} 6,237 \]

4.2.2 The outstanding balance can be calculated in 2 different ways:

**Method 1:** Present value or
**Method 2:** Future value

**Method 1:** Using the Present value formula
After 1 year (12 months):
\( P_v \; ; \; i = \frac{0.08}{12} \; ; \; n = 29 \times 12 = 348 \) remaining payments

The outstanding balance on the loan = the 'present value' (after 1 year) of the 348 remaining payments
\[ = \frac{x(1 + i)^n}{i} \]
\[ = \frac{6,237(1 + 0.08)^{348}}{0.08} \]
\[ = \text{R} 842,899,56 \]

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Method 2: Using the *Future value* formula

\[ A_{1y} = 850,000 \left(1 + \frac{0.08}{12}\right)^{12} \]

= 920,549.58 \ldots \text{value of loan after 1 year}

\[ F_v = \frac{6,237 \left(1 + \frac{0.08}{12}\right) - 1}{0.08} \quad \text{total payments made during the 1st year} \]

= R77,650.19

Outstanding Balance = 920,549.58 – 77,650.19

= R842,899.39 \checkmark

**Note:** The amount she paid during the 1st year, R77,650.19 = 12 × R6,237

**SECTION B**

5.1 Be meticulous in writing down the given expression correctly.

\[ f(n) = (\frac{12}{n})^{10} \times 440 \ldots \text{ } f(n) \text{ is the frequency of the } n^{th} \text{ key} \]

5.1.1 \[ f(73) = \left(\frac{12}{73}\right)^{10} \times 440 \]

= 1,760 Hz \checkmark

5.1.2 \[ f(n) = 3,520 \quad \Rightarrow \quad \left(\frac{12}{n}\right)^{10} \times 440 = 3,520 \]

\[ \Rightarrow \quad \left(\frac{12}{n}\right)^{10} = 8 \]

\[ \Rightarrow \quad n = 49 = \log_{12} 8 \]

\[ \Rightarrow \quad x = \log_b N \]

\[ = \frac{\log 8}{\log \frac{12}{2}} \]

= 36

\[ \Rightarrow \quad n = 85 \]

\[ \Rightarrow \quad \text{The } 85^{th} \text{ key } \checkmark \]

5.2.1 Note the difference between \( f^{-1}(x) \), the inverse of a function and \( f'(x) \), the derivative of a function.

**Equation of \( f \):** \( y = 3x - 1 \)

\[ \Rightarrow \quad \text{Equation of } f^{-1}: \quad x = 3y - 1 \]

\[ \Rightarrow \quad -3y = x - 1 \]

\[ \Rightarrow \quad y = 1 \frac{1}{3} x + 1 \frac{1}{3} \quad \text{i.e. } \]

\[ f^{-1}(x) = \frac{1}{3} x + 1 \frac{1}{3} \]

**Equation of \( g \):** \( y = 2^x \)

\[ \Rightarrow \quad \text{Equation of } g^{-1}: \quad x = 2^y \]

\[ \Rightarrow \quad y = \log_2 x, \quad \text{i.e. } \]

\[ g^{-1}(x) = \log_2 x \]

5.2.2 (i) \[ x \leq 1 \text{ or } x > 3 \ldots \text{ } f \text{ below } g \]

(ii) \[ 0 < x < 1 \ldots \text{ } g \text{ below the } x-axis \]

‘*Use the graph*’ means ‘read the solutions off the graph’. (Only 2 marks each.)

No complicated algebra is required.

(iii) \[ \text{Note: } f \text{ } \& \text{ } g \text{ intersect at } A(1; 2) \text{ } \& \text{ } B(3; 8). \]

\[ \Rightarrow \quad f^{-1} \text{ } \& \text{ } g^{-1} \text{ intersect at } (2; 1) \text{ } \& \text{ } (8; 3) \]

\[ \Rightarrow \quad x = 2 \quad \text{or } \quad 8 \]

**Confirm** by determining

\[ f^{-1}(x) \text{ } \& \text{ } g^{-1}(x) \]

for \( x = 2 \quad \& \quad x = 8 \).

(iv) \[ x > 2.1 \]

\[ h || f \quad \Rightarrow \quad h'(x) = f'(x), \quad \text{i.e. } h \text{ and } f \text{ have equal gradients} \]

Before point C (where \( x < 2.1 \)):

\[ g'(x) < h'(x) \]

At point C (where \( x = 2.1 \)):

\[ g'(x) = h'(x) \]

After point C (where \( x > 2.1 \)):

\[ g'(x) > h'(x) \quad \text{and so} \]

\[ g'(x) > f'(x) \]

5.3.1 At C:

\[ y = 2kx - k^2 \]

\[ \quad \Rightarrow \quad 2kx - k^2 = 2mx - m^2 \]

\[ \quad \Rightarrow \quad 2x(m - k) = k^2 - m^2 \]

\[ \quad \Rightarrow \quad x = \frac{k + m}{2} \]

\[ \Rightarrow \quad x_C = \frac{k + m}{2} \]

\[ \Rightarrow \quad \text{The solution of a linear literal equation in } x \text{ } \Rightarrow \text{ is required in this question. Make } x \text{ the subject! Note the factorisation required.} \]

6.1 You have been given the answer. Show how to arrive at the answer (5 marks!). [The given answer is not to be used for your calculation!]

\[ \text{The point } A(k; \ldots) \text{ lies on the graph } y = x^2 \]

\[ \Rightarrow \quad y_A = k^2 \]

\[ \Rightarrow \quad \text{Point } A = (k; k^2) \]

The gradient of the tangent to \( f \) for any \( x \) is \( 2x \ldots \text{ the derivative} \]

\[ \Rightarrow \quad \text{The gradient of the tangent to } f \text{ for } x = k \quad \Rightarrow \quad 2k \]

The equation of the tangent is:

\[ y = 2kx - k^2 \]

6.2 ‘*Hence write down*’ (for 1 mark) means use the previous finding (in Q6.1). *No working is required.*

\[ x_B = m \]

The process will be identical to that in 6.1.

\[ \Rightarrow \quad \text{The equation of the tangent to } f \text{ at } B \text{ is:} \]

\[ y = 2mx - m^2 \]

6.3 At C:

\[ y = 2kx - k^2 \quad \text{and} \quad y = 2mx - m^2 \]

\[ \Rightarrow \quad 2kx - k^2 = 2mx - m^2 \]

\[ \Rightarrow \quad 2kx - 2mx = k^2 - m^2 \]

\[ \Rightarrow \quad 2x(k - m) = k^2 - m^2 \]

\[ \Rightarrow \quad x = \frac{(k + m)(k - m)}{2(k - m)} \]

\[ \Rightarrow \quad x_C = \frac{k + m}{2} \]

\[ \Rightarrow \quad \text{The solution of a linear literal equation in } x \text{ is required in this question. Make } x \text{ the subject! Note the factorisation required.} \]

6.4 \[ \text{NB: } x_D = x_C \]

\[ \Rightarrow \quad C \text{ } \& \text{ } D \text{ lie on a vertical line, i.e. } \parallel \text{ } y-axis. \]

\[ x_D = \frac{x_A + x_B}{2} = \frac{k + m}{2} \quad \ldots \text{midpoint formula} \]

\[ \Rightarrow \quad x_D = x_C \]

\[ \Rightarrow \quad \text{CD is parallel to the } y-axis \]

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7.1 Again, as in question 6.1, you have been given the answer (to ensure that you proceed correctly for the question.) You need to SHOW how to arrive at this answer! (5 marks). [The given values are not to be used for your calculation in 6.1.]

The roots of the graph are: -6; -1 and 2

\[ \text{The equation of } f \text{ is: } y = a(x + 6)(x + 1)(x - 2) \]

Subst. \( F(0, 24) \):

\[ a(6)(1)(-2) = 24 \]
\[ a = -2 \]

Substitute \( a = -2 \):

\[ y = -2(x + 6)(x + 1)(x - 2) \]
\[ \therefore y = -2(x^3 + 7x - 12) \]
\[ \therefore y = -2x^3 - 14x^2 + 24x + 12 \]
\[ \therefore y = -2x^3 + 5x^2 - 8x - 12 \]
\[ \therefore y = -2x^3 - 10x^2 + 16x + 24 \]
\[ \therefore a = -2; b = -10; c = 16 \text{ and } d = 24 \]

7.2 At D (and at E):

\[ f'(x) = 0 \]

\[ -6x^2 - 20x + 16 = 0 \]

\[ -6(x^2 + \frac{10}{3}x - \frac{8}{3}) = 0 \]

\[ x = -4 \text{ and } x = \frac{2}{3} \] at \( E \)

\[ y_D = f(-4) = -2(-4)^3 - 10(-4)^2 + 16(-4) + 24 = -72 \]

\[ \therefore \text{ Point } D \text{ is } (-4; -72) \]

7.3 Write down the values of \( p \) and \( q \) (for 2 marks) as instructed. Excessive manipulation is not required. Use your understanding of horizontal translation.

The translation of point \( D(-4; -72) \) to \( O(0; 0) \) is:

\[ x: -4 \rightarrow 0 \quad \text{ ... } 4 \text{ units to the right} \]
\[ y: -72 \rightarrow 0 \quad \text{ ... } 72 \text{ units upwards} \]
\[ \therefore p = 4 \text{ and } q = 72 \]

8.1 Point B lies on \( f \) if \( x_B = x, \text{ then } y_B = f(x) \)

\[ \text{DB} \parallel y-axis \Rightarrow y_B = x_B = x \]

\[ B \text{ on the graph } y = 4 - \frac{x^2}{4} \Rightarrow y_B = 4 - \frac{x^2}{4} \]

\[ \therefore \text{ Point } B \text{ is } \left( x; 4 - \frac{x^2}{4} \right) \]

8.2 Again (as in Q6.1 and 7.1), do not work from the given answer.

\[ A = \frac{1}{2} \text{ OD, BD} \]
\[ = \frac{1}{2} x \left( 4 - \frac{x^2}{4} \right) \]
\[ = 2x - \frac{x^2}{8} \text{ units}^2 \]

The area of \( \Delta = \frac{1}{2} \text{ base } \times \text{ height} \)
\[ \text{, where OD = } x \text{ and the height, } BD = y_B = 4 - \frac{x^2}{4} \]

8.3 Note:

\[ A = 2x - \frac{x^2}{8} \]
\[ A' = - \frac{1}{2} \left( 3x^2 \right) \]
\[ \therefore A' = \frac{1}{3} \text{ of the coefficient, } \frac{2}{3} \text{ aside, at first} \]
\[ \frac{3}{2}, \frac{2}{3} \]

Maximum A occurs when \( A' = 0 \)

\[ 2 - \frac{3}{2}x^2 = 0 \]
\[ - \frac{3}{2}x^2 = -2 \]
\[ \times \left( \frac{3}{2} \right) \]
\[ x^2 = 16 \]
\[ x = \pm \frac{4}{\sqrt{3}} \]
\[ \therefore x_D > 0 \]
\[ \therefore x = \frac{4}{\sqrt{3}} \]
\[ x_D = 2.3 \quad \therefore D(2,3; 0) \]

\[ \text{D should be } 2.3 \text{ units from } O \]

8.4 Maximum \( A = 2(2.3) - \frac{(2.3)^3}{8} = 3.1 \text{ units}^2 \)

\[ \therefore \text{ or } \frac{16}{9} \text{ units}^2 \]

Check your calculator expertise!

9.1 The equation of \( f \):

\[ y = -x^2 + 4x \]

The x-intercepts:

\[ y = 0 \]
\[ x^2 - 4x = 0 \]
\[ x(x-4) = 0 \]
\[ x = 4 \quad \text{at } A \quad \text{ ... } x = 0 \text{ at } O \]

\[ \therefore A(4;0) \]

The turning point, \( E \):

\[ x_E = 2 \quad \text{ ... halfway between the roots} \]
\[ y_E = -2^2 + 4(2) = -4 + 8 = 4 \]

\[ \therefore E(2;4) \]

9.2 Again, (like Q8.2, 7.1 and 6.2), do not work from the given answer. Calculate \( x_C \) and \( y_A \) as indicated below.

\[ \text{OA} = 4 \quad \text{AB} = \frac{3}{4} \quad \text{of } 4 = 3 \quad \text{BC} = \frac{3}{4} \quad \text{of } 3 = \frac{9}{4} = \frac{21}{4} \]

\[ x_B = 7 \quad \text{and } \quad x_C = \frac{9}{4} \]

\[ \therefore x_C = \frac{7 + \frac{9}{4}}{2} = \frac{31}{8} = \frac{65}{8} \]

Sim., \( y_E = 4 \quad \therefore y_C = \frac{3}{4} \quad \text{of } 4 = 3 \quad \therefore y_D = \frac{3}{4} \quad \text{of } 3 = \frac{9}{4} = \frac{21}{4} \]

\[ \therefore G \left( \frac{65}{8}; \frac{21}{4} \right) \]

9.3 Equation of the 3rd parabola:

\[ y = a(x - p)^2 + q \]

\[ \therefore y = a \left( x - \frac{65}{8} \right)^2 + \frac{9}{4} \quad \text{ ... see turning pt. } G \text{ in 9.2} \]

Substitute \( (7;0) \):

\[ 0 = a \left( 7 - \frac{65}{8} \right)^2 + \frac{9}{4} \]

\[ a = \frac{9}{4} \]

\[ a = \frac{9}{4} \quad \text{at } \frac{81}{8} \]
\[ \times 64 \quad \therefore a = \frac{9}{4} \text{ at } 64 \]
\[ \times 81 \quad \therefore a = \frac{9}{4} \text{ at } 81 \]
\[ \therefore a = \frac{16}{9} \]

Equation:

\[ y = -\frac{16}{9} \left( x - \frac{65}{8} \right)^2 + \frac{9}{4} \]

9.4 \( S_x \) of \( OA + AB + BC + CD + \ldots \)

\[ = 4 + 3 + \frac{9}{4} + \ldots \]

\[ S_x \text{ of a G.S. with } a = 4, \text{ and } r = \frac{3}{4} \]

\[ = 4 \left( 1 - \frac{3}{4} \right) \quad \therefore S_x = \frac{a}{1 - r} \]

\[ = 4 \times \frac{1}{1 - \frac{3}{4}} \]
\[ = 16 \]

\[ \therefore \] All the parabolas won’t fit on OH because \( OH = 15 \), which is < \( 16 \)
10.1 If A and B are independent events:
\[ P(A \cap B) = P(A) \cdot P(B) \]
\[ \therefore 0.1 = (x + 0.1)(0.1 + 0.3) \]
\[ \therefore 0.1 = (x + 0.1)(0.4) \]
\[ + 0.4 \]
\[ \therefore 0.25 = x + 0.1 \]
\[ \therefore x = 0.15 \]

\& \ y = 1 - (x + 0.1 + 0.3) \ldots \text{the complement of } A \cup B
\[ = 1 - 0.55 \]
\[ = 0.45 \]

10.2.1 \[ P(\text{male/fail}) = \frac{8}{98} = \frac{4}{49} = 0.08 \]
\[ \ldots \#1 \]

10.2.2 Given female, \[ P(\text{pass}) = \frac{43}{58} = 0.74 \]
\[ \ldots \#2 \]

11.1 \[ \text{There are 8 letters in the word, but the first letter is fixed. If the remaining 7 letters were all different, then the number of arrangements is } 7! = 5,040. \]

\[ \text{But, the e occurs twice} \]
\[ \therefore \text{The number of arrangements is } 7! \div 2! = 2,520 \]

Note: The arrangements include duplication because of e occurring in 2 spots.
- The number of arrangements of n items = n! ;
- The number of arrangements of n items where r are identical = \( \frac{n!}{r!} \)

11.2 The total number of ways in which 5 people could be lined up is 5! = 120.

For Piet to be next to Sipho and Jabu next to Jane, it is as though there are now 3 people that can be lined up in 3! = 6 ways, BUT each pair could be in a different order, \( \therefore 2! \) ways for each pair.

\[ \therefore \text{Conditional on the stated pairing, the number of ways the 5 could be lined up is } 3! \times 2! = 24. \]

\[ \therefore \text{The probability that both pairs 'happen'} = \frac{24}{120} = \frac{1}{5} \]

1.1.1 \[ \text{Estimated mean, } \mu = 19.5 \]
\[ \therefore B \]

1.1.2 \[ \text{Estimated standard deviation, } \sigma = 10.712... \]
\[ \therefore D \]

2.2 \[ \text{The equation of the regression line: } y = A + Bx \]
\[ A = 28.14115058... \]
\[ B = 0.08857785071... \]
\[ \therefore \text{The eqn.: } y = 28.1412 + 0.0886x \]
\[ \ldots \text{correct to 4 dec. digits} \]

2.3 \[ \text{The charge } = \text{ the flat call out fee } + R50 \times \text{ the number of half hours} \]

\[ \text{This is what is called a STEP FUNCTION} \]

If the area, \[ x = 560 \text{ m}^2 \]
then \[ y = 28,314.3 + 0.0884 \times 560 \]
\[ = 77,818.3 \text{ minutes} \]
\[ + \frac{30}{60} = 2,593.9 \ldots \text{half-hours} \]
\[ = 3 \text{ half-hours} \]

\[ \therefore \text{The charge } = R150 + R50 \times 3 = R150 + R150 = R300 \]

2.4 \[ \text{The area, } x (\text{m}^2) = 100 \text{ m} \times 70 \text{ m} = 7,000 \text{ m}^2 \]

According to the regression equation:
\[ \text{The time, } y \text{ (in minutes) } = 28,314.3 + 0.0884 \times 7,000 \]
\[ = 605.114 \ldots \text{minutes} \]

But, no, one should not use the regression equation because the value of x is too far outside the values used to establish the regression line.

3.1.1 \[ x_k = \frac{4 + 12}{2} = 8 \ldots K \text{ midpoint } AS \]
\[ \& y_k = y_A = y_S \ldots AKS || x-axis \]
\[ = 16 \]
\[ \therefore \text{Equation of } OK: \]
\[ y = -\frac{4}{3}x \]

3.1.2 \[ m_{OS} = \frac{16}{12} = \frac{4}{3} \ldots AT \perp OS \]
\[ \therefore m_{AT} = -\frac{3}{4} \]

Substitute \[ m = \frac{3}{4} \text{ and } A(4; 16) \] in
\[ y = mx + c \]
\[ \therefore 16 = \left( -\frac{3}{4} \right) (4) + c \]
\[ \therefore c = 19 \]

\[ \therefore \text{Equation of AT:} \]
\[ y = -\frac{3}{4}x + 19 \]

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4.1 Gradient of diameter = 2

\( \text{Gradient of tangent} = -\frac{1}{2} \)

Equation of tangent:

Subst. \( m = -\frac{1}{2} \) & \( F(12; 0) \) in

\[
\begin{align*}
y - 0 &= m(x - 12) \\
y &= -\frac{1}{2}x + 6
\end{align*}
\]

At \( E \): \( y = -\frac{1}{2}x + 6 \) and \( y = 2x + 3 \)

\( E \) is the point of intersection of the 2 lines.

4.2

The radius of \( \odot C \) = 3

& \( \odot B \)

The notation \( K_1 \) is to be used rather than \( SKO \).

4.3 To convert to standard form, complete the square...

\[
x^2 + 4\cos \theta x + y^2 + 8\sin \theta y = -3
\]

add \( \left( \frac{1}{2} \right) \) the coefficient of \( x \) & \( \left( \frac{1}{2} \right) \) the coefficient of \( y \):

\[
x^2 + 4\cos \theta x = (2\cos \theta)^2 \quad y^2 + 8\sin \theta y + (4\sin \theta)^2 = 4\cos \theta \sin \theta + 16\sin^2 \theta
\]

The coefficient of \( x \) in \( 4\cos \theta \) is \( 4 \cos \theta \); the coefficient of \( y \) in \( 8\sin \theta \) is \( 8 \sin \theta \).

\[
r^2 = -3 + 4(1 - \sin^2 \theta) + 16\sin^2 \theta = 4\cos \theta \sin \theta + 16\sin^2 \theta = 1 + 12\sin^2 \theta
\]

Note:

For any value of \( \theta \): 

\( -1 \leq \sin \theta \leq 1 \)

\( \quad \leq \sin^2 \theta \leq 1 \)

\( \quad \leq 12\sin^2 \theta \leq 12 \)

\( + 1) \quad \leq 12\sin^2 \theta + 1 \leq 13 \)

\( \quad \) the maximum value of \( 12\sin^2 \theta + 1 \) is 13

i.e. the maximum value of \( r^2 \) is 13

\( \quad \) the maximum value of \( r \) is \( 2\sqrt{13} \) units.
5.1.2 \[ \tan \left( \frac{2\theta + 100^\circ}{3} \right) = \tan 311,066^\circ = -1,148 \]

5.2 \[ \frac{+ \sin A}{(-\sin A)} = \frac{\sin A}{-2 \sin A} = \frac{1}{2} \]

5.3.1 \[ \tan \beta = \frac{k}{6} \quad \ldots \quad \tan \beta = \frac{y}{x} \] by definition \[ \therefore \frac{2}{4} = \frac{1}{2} \times 8) \quad \therefore k = 2 \]

5.3.2 \[ OT^2 = 8^2 + 2^2 \]
\[ \therefore OT = 8 \]
\[ \therefore OT = \sqrt{68} = 4 \times \sqrt{17} = 2 \sqrt{17} \]
\[ \therefore \sin \beta = \frac{2}{2 \sqrt{17}} = \frac{1}{\sqrt{17}} \quad \text{(or:} \quad \frac{1}{\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}} = \frac{1}{17} \))

5.4 \[ \text{Expr.} = \cos 45^\circ \cos \theta + \sin 45^\circ \sin \theta - \frac{\sin \theta}{\cos \theta} \]
\[ = \sqrt{2} \left( \frac{1}{17} \cos \theta + \frac{1}{17} \sin \theta \right) - \frac{\sin \theta}{\cos \theta} \]
\[ = \frac{\cos \theta + \sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \]
\[ = \cos \theta + \sin \theta - \sin \theta \]
\[ = 1 \]

6.1.1 \[ 120^\circ < \ldots \text{1 full wave from 0}^\circ \text{to 120}^\circ \]

6.1.2 \[ \text{The fraction of the 'normal' period (for } y = \cos x) \]
\[ = \frac{120^\circ}{360^\circ} = \frac{1}{3} \]
\[ \therefore a = 3 \quad \ldots \text{3 waves over 360}^\circ \]

6.1.3 \[ 60^\circ < \ldots \text{1 full wave from 0}^\circ \text{to 60}^\circ \]

6.1.4 \[ \text{The fraction of the 'normal' period (for } y = \tan x) \]
\[ = \frac{60^\circ}{180^\circ} = \frac{1}{3} \]
\[ \therefore b = 3 \]

6.2 At P: \[ x = 60^\circ \]
\[ \& y = h(60^\circ) = 4 \tan 60^\circ = 4 \left( \frac{\sqrt{3}}{1} \right) = 4 \sqrt{3} \]
\[ \therefore P (60^\circ; 4 \sqrt{3}) \]

6.2.2 At Q: \[ f(x) = g(x), \text{ i.e. } 2 \sin x = 1.5 \cos x \]
\[ + 2 \cos x \quad \therefore \tan x = 0.75 \quad \ldots \quad \frac{\sin x}{\cos x} = \tan x \]
\[ \therefore x = 36,869^\circ \]
\[ \& y = f(36,869^\circ) = 2 \sin 36,869^\circ \]
\[ \text{or } y = g(36,869^\circ) = 1.5 \cos 36,869^\circ = 1.2 \]
\[ \therefore Q(36,87^\circ; 1.2) \quad \ldots \text{correct to two decimal digits} \]

7.1 \[ P = \theta - \alpha \quad \ldots \text{ext. } \angle \text{ of } A = \text{sum. of int. opp. } \angle \]

7.2 \[ \text{Note: In non-right } \Delta \text{, we have the choice of 2 tools: sine rule or cosine rule. Let's try sine rule . . .} \]

In \( \Delta \text{RSQ}: \)
\[ \text{SRQ} = \theta \quad \text{base } \angle \text{ of isos. } \Delta \text{QRS} \]

Did you notice that \( \Delta \text{QRS} \) was isosceles?

In \( \Delta \text{RPQ}: \)
\[ \sin (\theta - \alpha) = \sin (\theta + \alpha) \]
\[ \times 2) \quad 2 \sin \theta \cos \alpha - \cos \theta \sin \alpha = \sin \theta \cos \alpha + \cos \theta \sin \alpha \]
\[ 2 \sin \theta \cos \alpha - 2 \cos \theta \sin \alpha = \sin \theta \cos \alpha - \cos \theta \sin \alpha \]
\[ \sin \theta \cos \alpha = 3 \cos \theta \sin \alpha \]
\[ + \cos \theta \cos \alpha \]
\[ \sin \theta \cos \alpha \cos \theta \sin \alpha \]
\[ = \frac{3 \cos \theta}{\cos \theta} \cdot \sin \alpha \]
\[ \therefore \tan \theta = 3 \tan \alpha \]

8.1 \[ (3x)^2 = 5^2 + (2x)^2 - 2(5)(2x) \cos L \]
\[ 9x^2 = 25 + 4x^2 - 20x \cos L \]
\[ 20x, \cos L = 25 - 5x^2 \]
\[ \cos L = \frac{5(1 - x^2)}{20x} \]
\[ \therefore \cos L = \frac{5 - x^2}{4x} \]

8.2 \[ \cos L < 0 \text{ if } \hat{L} \text{ is obtuse} \]
\[ \text{but the minimum value of } \cos L = -1 \]
\[ -1 \leq \cos L < 0 \]

8.3 \[ \text{If } x = 6, \cos L = \frac{5 - 36}{24} = \frac{31}{24} \quad (<1) \]
\[ \therefore \text{No, not possible} \quad \ldots \text{see 8.2} \]

8.4 \[ \text{Area of } \Delta \text{LMN} = \frac{1}{2} (5)(2x) \sin L \]
\[ \text{Now, if } x = 3, \cos L = \frac{5 - 32}{4(3)} = -\frac{4}{3} \]
\[ \therefore \hat{L} = 180^\circ - 70,53^\circ = 109,47^\circ \]
\[ \text{Area of } \Delta \text{LMN} = \frac{1}{2} (5)(6) \sin 109,47^\circ \quad \ldots \text{if } x = 3 \]
\[ = 14,1 \text{ square units} \]

9.1

9.2

9.3

10.2.1 \[ \text{AV} = AD \quad \ldots \text{tangents from a common point} \]
\[ \therefore \hat{V}_3 = \hat{D}_2 \]
\[ \text{base } \angle \text{ of isosceles } \Delta \text{AVD} \]
\[ \text{In } \Delta \text{AVD: } \hat{V}_3 + \hat{D}_2 = 140^\circ \quad \ldots \angle \text{ of } \Delta; \hat{A}_1 + \hat{A}_2 = 40^\circ \]
\[ \therefore \hat{V}_3 = \hat{D}_2 = 70^\circ \]
\[ \therefore \hat{D}_1 = 90^\circ - 70^\circ \ldots \text{ADO} = 90^\circ \text{in 10.1.1} \]
\[ = 20^\circ \]

Tip: Fill the \( \angle \) in on the sketch as you go.
12.1 Reasons:

\[ \triangle \text{LE} \parallel \triangle \text{AD} \]

alternate \( \triangle \)

\[ \triangle \text{LE} \ parallel \ triangle \text{AD} \]

12.2.1 \( \hat{N} = 90^\circ \) . . . \( \hat{N} \) in semi-circle

\[ \triangle \text{AN} = 16 \text{ units} \]

12.2.2 In \( \triangle \text{NAD} \):

\[ \frac{\text{AL}}{\text{AN}} = \frac{\text{DE}}{\text{DN}} \]

\[ \triangle \text{LE} \parallel \triangle \text{AD} \]

proportion thm.

But \( \triangle \text{DE} = \triangle \text{AE} \) (= 20 units) . . . proved in 12.1

\[ \frac{\text{AL}}{\text{16}} = \frac{20}{32} \]

\[ \text{DN} = \text{DE} + \text{NE} = 20 + 12 \]

\[ \frac{\text{AL}}{\text{16}} = \frac{20}{32} \times 16 \]

\[ = 10 \text{ units} \]

13.1 Reasons:

\( \triangle \) in the same segment, subtended by chord RD in the small circle

\[ \tan (RE) \] chord (ED) theorem

\[ \triangle \text{E} \]

13.2 \( \hat{O}_1 = \hat{E}_1 + \hat{A}_2 \) . . . exterior \( \hat{\triangle} \) of \( \hat{\Delta} \)

But \( \hat{A}_2 = \hat{E}_2 \) . . . see 13.1

\[ \hat{O}_1 = \hat{E}_1 + \hat{E}_2 \]

13.3 In \( \triangle \text{EDR} \) and \( \triangle \text{AER} \)

(1) \( (\hat{R}_1 + \hat{R}_2) \) is common

(2) \( \hat{E}_2 = \hat{A}_2 \) . . . see 13.1

\[ \triangle \text{EDR} \parallel \triangle \text{AER} \]

\[ \triangle \text{EDR} \parallel \triangle \text{AER} \]

\[ \triangle \text{EDR} \parallel \triangle \text{AER} \]

Note: Choose the sides mentioned in the question.

14.1 \( \triangle \text{ABD} \parallel \triangle \text{CBA} \parallel \triangle \text{CAD} \)

14.2.1 \( \triangle \text{ABD} \parallel \triangle \text{CBA} \parallel \triangle \text{CAD} \)

\[ \triangle \text{ABD} \parallel \triangle \text{CBA} \parallel \triangle \text{CAD} \]

\[ \text{AB}^2 = \text{BC} \cdot \text{BD} \]

14.2.2 \( \triangle \text{ABD} \parallel \triangle \text{CAD} \parallel \triangle \text{CBD} \)

\[ \text{AD}^2 = \text{CD} \cdot \text{BD} \]

14.3 Hence: \( \text{AB}^2 + \text{AD}^2 = \text{BD} \cdot \text{BC} + \text{CD} \cdot \text{BD} \)

\[ = \text{BD} \cdot \text{BC} + \text{CD} \cdot \text{BD} \]

\[ = \text{BD} \cdot \text{BC} + \text{CD} \cdot \text{BD} \]

\[ = \text{BD} \cdot \text{BC} + \text{CD} \cdot \text{BD} \]

\[ = \text{BD} \cdot \text{BC} + \text{CD} \cdot \text{BD} \]

You have just proved the Theorem of Pythagoras, using similar \( \triangle \)!

This example also illustrates that: The perpendicular from the right \( \hat{\triangle} \) to the hypotenuse produces 3 similar \( \triangle \).